QM4226
QUANTUM THEORY : REVISION OF SEC.1

1. Show that the time-evolution operator $T(t, t_0)$ obeys an equation similar in form to the TDSE (eq 1.5).

2. derive the form of time-evolution operator for infinitesimal time-intervals , $\exp -\frac{i}{\hbar}\hat{H}(t)\delta t$ (eq 1.12).

3. Prove that $T(t, t_0)$ is unitary.

4. Prove that unitarity of T implies conservation of probability.

5. If a quantum system is in a quantum state $\Psi(x) = \sum_n C_n|n>$ which is a superposition of orthonormal eigenstates $|n>$ of an operator A, with eigenvalues a_n, what experimentally significant quantity do you obtain from $C_n =< n|\Psi >$?

6. Write $< n|m >$ as an integral over x.

7. Give $< A >=< \psi A \psi >$ in terms of C_n and a_n. What is the physical significance of $< A >$ in an experiment?

8. For the above system what is $e^A|n >$ equal to?

9. Hence what is $e^A\Psi$ equal to? What is $< \exp A >$ equal to ?

10. Explain the observed phenomena of wavepacket 'revivals' in a time-independent system. Discuss physical significance of $P(x, t)$ (eq.1.25) vs $|C_n|^2$ in the superposition. Calculate t_R.

11. Show that a free gaussian wavepacket spreads continuously in time (eq.1.28-30) for derivation in position representation or exercise (1.3) for momentum representation.

12. Explain how a gaussian wavepacket(GWP) satisfies the Heisenberg Uncertainty principle (eg multiply the widths of $|\psi(x)|^2$ and $|\Phi(p)|^2$). For the most accurate treatment you should be able to calculate variances, eg evaluate $(\Delta x)^2 =< (x- < x >)^2 >$. You are not expected to memorise gaussian integral formulae.

13. Derive the closure relation (Appendix I.5).

14. What are the eigenfunctions $|x >$ and $|p >$ of the operators $\hat{P}$ and $\hat{x}$ respectively? Give $< x|p >$, $< x|\Psi >$ and $< p|\Psi >$, $< p|p' >$ in non-Dirac notation. Write them all also as overlap integrals, eg $< x|\Psi >= \Psi(x) =_{-\infty}^{\infty} \int \delta(x - x')\Psi(x')dx'$.

15. Explain the split-operator method for propagating a quantum wavefunction with time. To what order is it accurate? Consider also the alternative formulation (to the one in the notes) $T(t + \delta t, t) \simeq \exp -\frac{i}{\hbar}\hat{H}_p\delta t/2. \exp -\frac{i}{\hbar}\hat{V}\delta t. \exp -\frac{i}{\hbar}\hat{H}_p\delta t/2$ to explain how you might propagate a wavefunction in time.

16. Describe a non-perturbative (Floquet!) method for calculating the time-evolution of a quantum system in a time-periodic Hamiltonian of period τ. Why might you expect the eigenvalues of $T(\tau, 0)$ to be complex (is T Hermitian?)?

17. What does $< nAm >$ represent? Explain how you obtain eigenvalues and eigenvectors of a quantum operator by a matrix method.