QM4226
QUANTUM THEORY : REVISION OF SEC.2

1. Explain Dirac's method of variation of constants (used for a quantum system with Hamiltonian $H = H_0 + \lambda V(x,t)$ where we know the solution with $V(x,t) = 0$) and show that it leads to a set of 1st order differential equations, for the coefficients, of the form:
   $dC_k/dt = \frac{\lambda}{i\hbar} \sum V_{kn}(t) e^{i\omega_{kn}t} C_n(t)$ Explain the meaning of all the symbols.

2. Show that, if the system is subjected to a perturbation $\lambda V(x,t)$, where λ is small, to first order, the probability amplitude equals $C_k + C_k^1$ and for a system with $C_k = \delta_{ki}$ at $t = 0$, the 1st order correction is given by:
   $C_k^1(t) = \frac{\lambda}{i\hbar} {}_0^t \int V_{ki}(t') e^{i\omega_{ki}t'} dt'$ Explain all the symbols.

3. Show that the first order correction to the amplitude implies no loss of probability for the initial state i.

4. Given the above equation for the first order terms, show that even for $V_{ki} = 0$, the $k - th$ and $i - th$ states may be coupled to second order via an intermediate set of states n. How weak is this effect likely to be, relative to 1st order?

5. For a system subjected to a harmonic perturbation $V \cos \omega t$, show that the first order transition probabilities are strongly peaked around energies corresponding to $\hbar\omega_{ki} = \pm\hbar\omega$.

6. Consider the perturbation corrections, to first order, for a system where we *do not* assume that at $t = 0$ (or before we turn on the time-dependent perturbation) $C_k = \delta_{ki}$. For example, calculate the first-order probabilities, for the various examples in the lectures, if we start from an initial state with, eg $C_k = \sqrt{\frac{1}{3}}\delta_{ki} + \sqrt{\frac{2}{3}}\delta_{ki'}$.