QM4226
QUANTUM THEORY &
REVISION of SECTION 4

Given $[J_x, J_y] = i\hbar J_z$ (including cyclic permutations) and $[J^2, J_i] = 0$ for $i = x, y, z$ show other commutation relations Note that below, for convenience, I do not have a 'hat' on the operators.**YOU** though, should take care to distinguish between an operator $\hat{j}$ and its corresponding quantum number j. eg the eigenvalue equation $\hat{j}|jm>= j(j+1)|jm>$.

1. Show $[J^2, J_\pm] = 0$
2. Show $J_+J_- = J^2 - J_z^2 + \hbar J_z$
3. Prove that $J_\pm|jm \pm 1>= \sqrt{j(j+1) - m(m \pm 1)}\hbar|jm \pm 1>$
4. $[J_+, J_-] = 2\hbar J_z$
5. $[J_z, J_\pm] = \pm\hbar J_\pm$
6. Then if $J = j_1 + j_2$ and $[j_{1k}, j_{2n}] = 0$ while $[j_{1k}, j_{1l}] = i\hbar j_{1n}$, where k, l, n represent a cyclic permutation of x, y, z, show
7. $2j_1.j_2 = J^2 - j_1^2 - j_2^2$
8. and $j_{1+}j_{2-} + j_{2+}j_{1-} = J^2 - j_1^2 - j_2^2 - 2j_{1z}j_{2z}$
9. $[J^2, j_1^2] = [J^2, j_2^2] = 0.$
10. and prove $[J^2, J_z] = 0$ but $[J^2, j_{1z}] \neq 0$ and $[J^2, j_{2z}] \neq 0$
11. NOW for extra practice, repeat all the above for the case where j_1, j_2 correspond to orbital and spin angular momentum respectively, ie with $J = L + S$ (eg instead of $[j_{1k}, j_{2j}] = 0$, show $[L_x, S_y] = 0$ etc. The maths is clearly the same.
12. Given Clebsch-Gordan coefficients,$< j_1m_1j_2m_2|JM >$ can you construct states $|j_1j_2JM >$ which are simultaneous eigenfunctions of J^2, j_1^2, j_2^2, J_z in terms of $|j_1m_1 >, |j_2m_2 >$?
13. Repeat this for the spin states $|SM >$ as shown in Equations. 4.68- 4.70.
14. Practice obtaining expectation values of *any* operator using $|JM >$ or superpositions of $|j_1m_1 >, |j_2m_2 >$ (see Homework 4 for examples).
15. Revise Spin thoroughly: all the material from equation (4.26) through to (4.72). Test yourself and show that:
16. $S_y|\alpha >= i\hbar/2|\beta >$
17. $S_x^2|\chi >= \frac{\hbar^2}{4}|\chi >$
18. if $|\chi >= |\alpha >$, what is $< S_x >$?
19. An electron is acted on by an operator $A = \sigma_x.\sigma_y$. Give this in matrix form, using $|\alpha >$ and $|\beta >$ as a basis. Verify Eq.(4.46). If $|\chi >= |\alpha >$, what is $< A >$?