QM4226
QUANTUM THEORY : REVISION OF SEC.5

Knowing the form for $\nabla = \hat{r}\frac{\delta}{\delta r} + \hat{\theta}\frac{1}{r}\frac{\delta}{\delta\theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\delta}{\delta\phi}$ and

$\nabla^2 = \frac{1}{r^2}\frac{\delta}{\delta r}(r^2\frac{\delta}{\delta r}) + \frac{L(\theta,\phi)}{r^2}$

1. Obtain the current $J = Re(\psi^* \frac{\hbar}{mi}\nabla\psi))$ for a plane wave $\exp i\mathbf{k}.\mathbf{r}$ and spherical wave $\frac{\exp ikr}{r}$. for large r.
2. Show that the asymptotic scattering solution (incident plane wave plus spherical waves) solves the TISE as $r \to \infty$.
3. Prove that $\frac{\delta\sigma}{\delta\Omega} = |f(\theta,\phi)|^2$.
4. Given Eq. (5.33) derive eq (5.36): $f(\theta,\phi) = \sum_l (2l+1)\frac{S_l(k)-1}{2ik}P_l(\cos\theta)$.
5. Using orthogonality of Legendre Polynomials, from [4], obtain $\sigma_{el} = \frac{\pi}{k^2}\sum_l (2l+1)|S_l(k)-1|^2$.
6. Obtain a similar form for the inelastic cross section. Hence derive the optical theorem.
7. Given $\sigma_{el} = \frac{\pi}{k^2}\sum_l (2l+1)\sin^2\delta_l$, explain (a)resonances in a partial wave (b) resonances (for s-wave dominated scattering: can have $\delta_l = \pi$). What is the scattering length?
8. By expanding a scattering length about $E = E_r$, derive the Breit-Wigner form for a resonance, Eq.(5.57).