Basic Ideas

3C24
Nuclear and Particle Physics

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(based on Lecture Notes from Ruben Saakyan)
UCL

HISTORY

19th century: atoms indivisible

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1897: Thomson's discovery of e^-

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1900: Photon (γ) postulated by Planck to explain black-body radiation

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1911: Rutherford experiments – central nucleus(+) orbited by e^-'s

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Bohr model of atom: first window into quantum physics

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1930: Neutrino (ν) postulated by Fermi to save energy conservation in β decay

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Nucleus is composite. Nucleons: protons (+e) and neutrons

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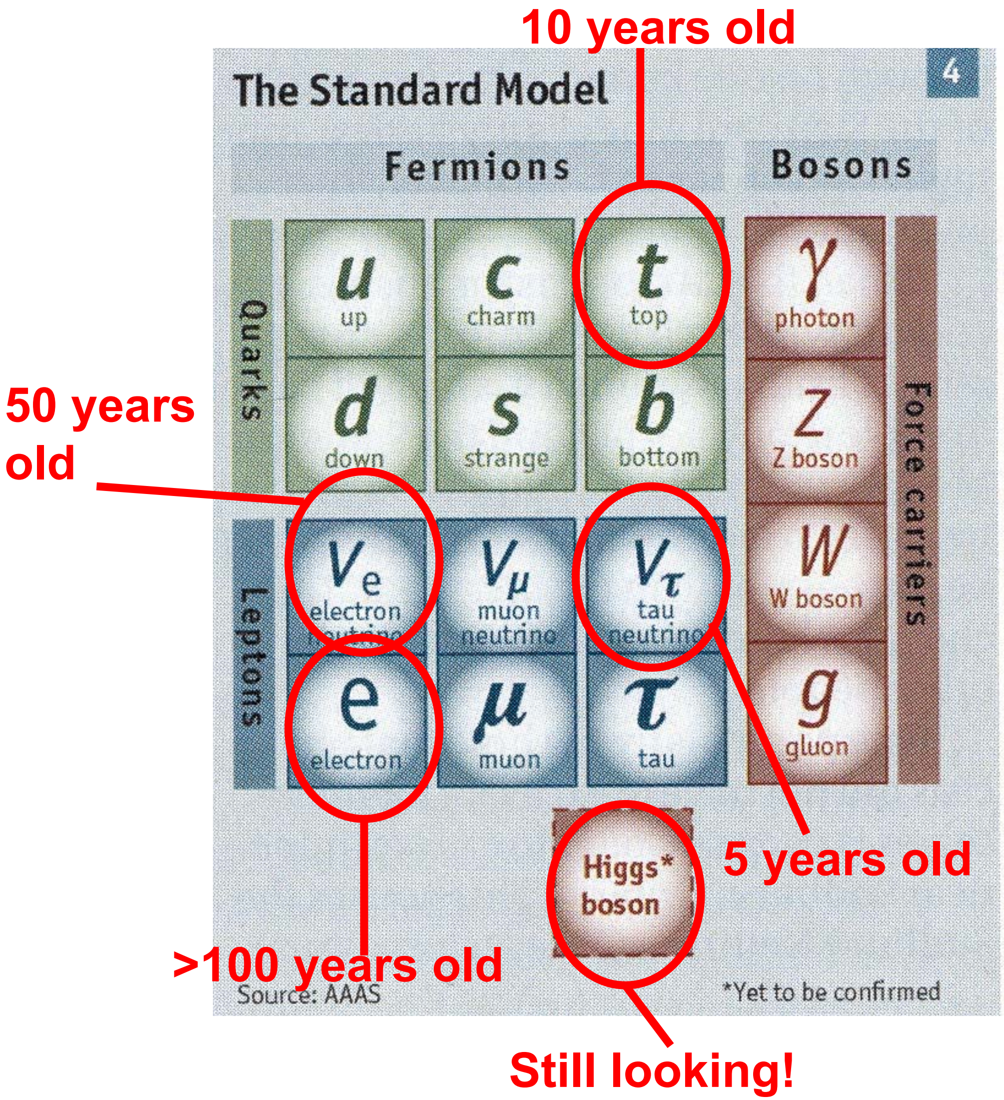
1960's: Nucleons are bound states of quarks which have fractional electrical charges (-e/3, 2e/3)

The Standard Model



Fundamental interactions:

- ✓ Electromagnetic—γ
- ✓ Strong –g (gluons)
- ✓ Weak—W and Z
- ✓ Gravitational—graviton?

Other non-elementary important Particles: Hadrons

Baryons	Mesons
p – uud	$\pi^+ = u\bar{d}$
n – udd	$K^0 = d\bar{s}$
Λ - uds	$\psi = c\bar{c}$

Relativity and antiparticles

- **$E=mc^2 \Rightarrow$ high energies are required to produce new particles**
- **$\lambda=h/p \Rightarrow$ high energies are needed to study small things**
  - **Proton radius ~ 10^{-15}m,**
  - **$>10^3 \times m_e$ energy needed**
- **relativistic effects are important!**
- **Quantum Theory must work with Special relativity $\Rightarrow$ ANTIPARTICLES**
  - **each particle must have an antiparticle**
  - **Symmetry between particles and antiparticles**
  - **opposite quantum numbers: electric charge, spin, lepton charge, etc…**
- **Particle+Antiparticle$\rightarrow$ annihilation (γ's or other)**

Units: Length, mass and energy

- Length, cross-sections
  - Tiny distances – fm (10^{-15} m)
  - Proton radius ~ 0.8 fm, range of nuclear force ~ 1-2 fm range of weak force ~ 10^{-3} fm
  - Cross-section: Barn. 1 b = $10^{-28}m^2$.
  - $\sigma(pp)$ ~ n×10 mb (~$10^{-30}m^2$) – strong interaction
  - $\sigma(\nu p)$ ~ n×10 fb (~$10^{-42}m^2$) – weak interaction
- Mass, Energy: electron volt 1 eV = 1.6×10^{-19} joules
  - 1 keV = 10^3 eV, 1MeV = 10^6 eV natural radioactivity, ^{238}U,^{232}Th
  - 1 GeV = 10^9 eV, 1 TeV = 10^{12} eV accelerators
  - 1 PeV = 10^{15} eV ultra high energy cosmic rays

Natural Units

- In order to create a particle of mass *M*, we need an energy at least as great as its rest energy $E = Mc^2$
- "Typical" masses
  - M_e = 0.511 MeV/c^2, M_p = 0.938 GeV/c^2, M_n = 0.939 GeV/c^2, M_W = 80.3 GeV/c^2, M_Z = 91.2 GeV/c^2
- Particle physics convention: Natural units

$\hbar \equiv h/2\pi = 1$ and $c = 1$ ⟶ M_e = 0.511 MeV

(BTW, it is only 9×10^{-28} g)

Particle Reactions

Typically, particle physicists smash particles together, then watch what comes out.

$e^- + p \rightarrow e^- + p$ elastic scattering

- Same particles in and out

$\nu_e + n \rightarrow e^- + p$ inelastic scattering

- Different particles out

The same initial particles can lead to different final states:

$e^- + e^- \rightarrow e^- + e^-$

$e^- + e^- \rightarrow \gamma + \gamma$

Particle Reactions. Decay.

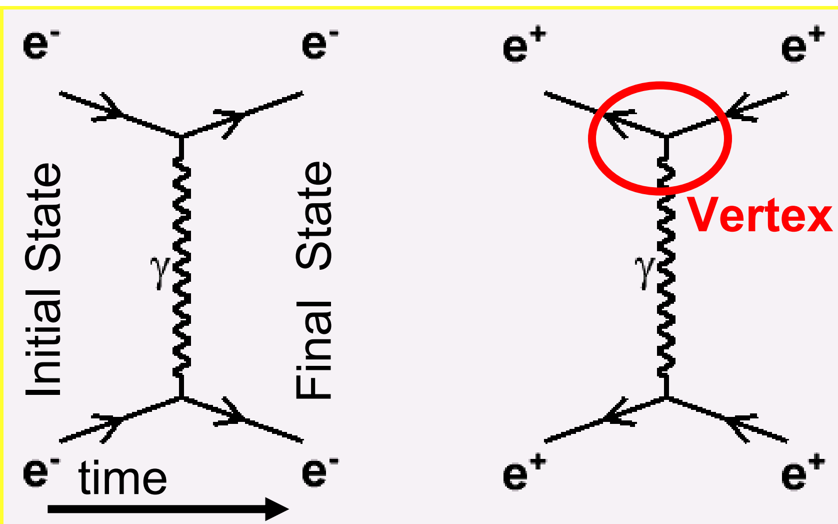
- β decay of neutron
  - $n \rightarrow p + e^{-} + \overline{\nu}_e$
  - N(t) = $N_0 \times \exp(-t/\tau)$, $\tau \approx 900$ sec
- β decay of a nucleus
  - $(Z, A) \rightarrow (Z+1, A) + e^{-} + \overline{\nu}_e$
  - Z – number of protons (charge)

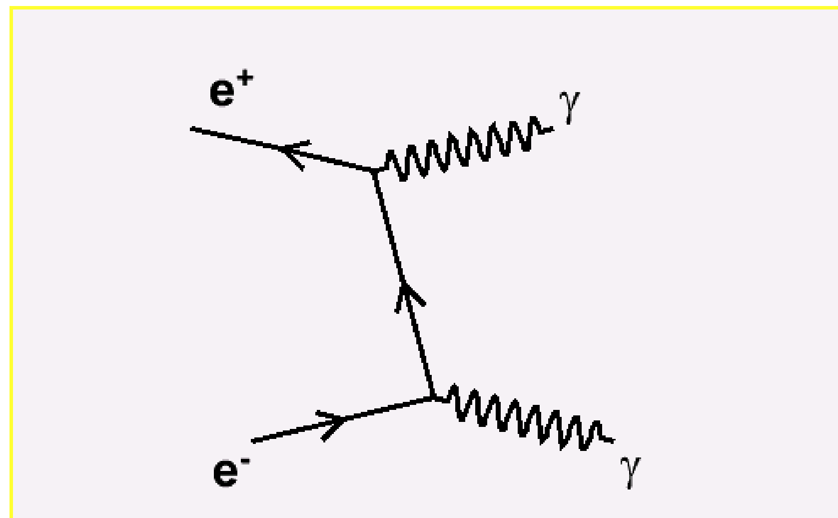
    A – number of neutrons + protons (mass number)

Feynman diagrams

A pictorial representation of a mathematics formalism that allows you to calculate probabilities for reactions

Electromagnetic interactions



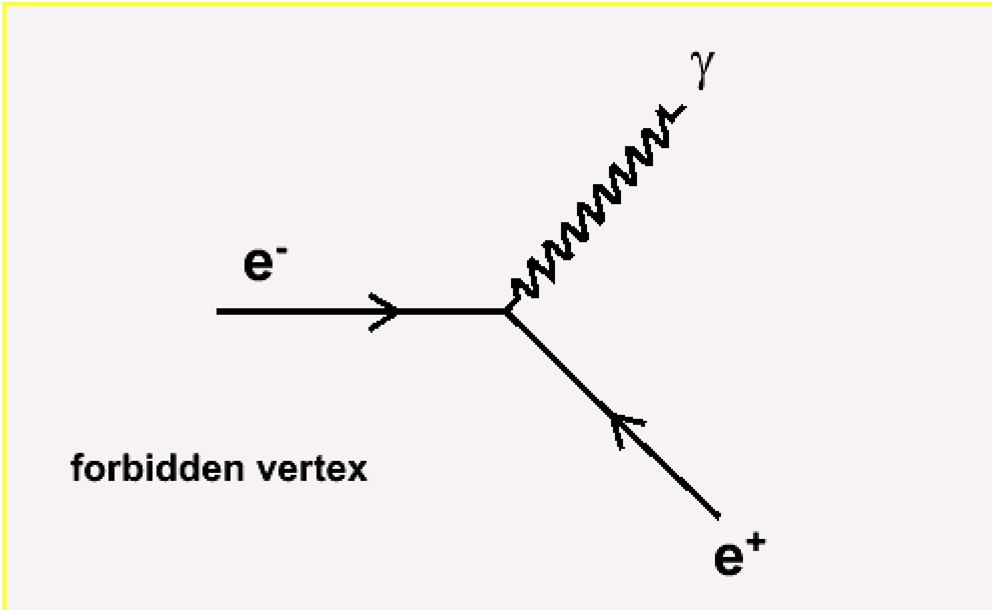


$e^- + e^- \rightarrow e^- + e^-$ $e^+ + e^+ \rightarrow e^+ + e^+$ $e^+ + e^- \rightarrow \gamma + \gamma$

The arrows do **NOT** indicate the direction of motion

They are for particles (→) and antiparticles (←)

Charge must be conserved at vertex



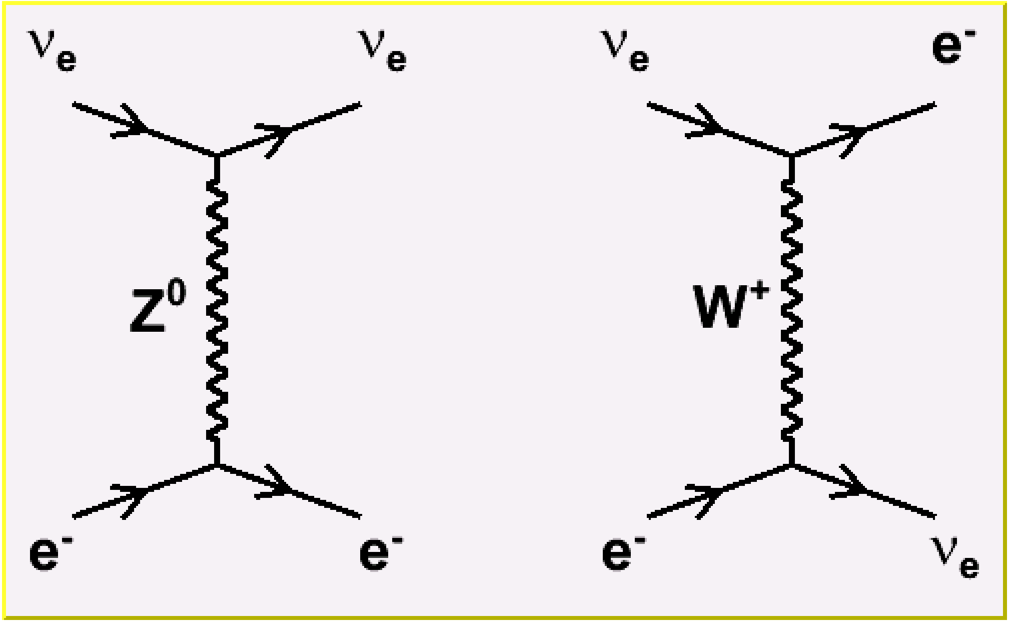
Forbidden process $e^- \rightarrow e^+ + \gamma$
(charge conservation violated)

Other rules apply at the vertex—these will come in later lectures

Feynman diagrams

Weak interactions

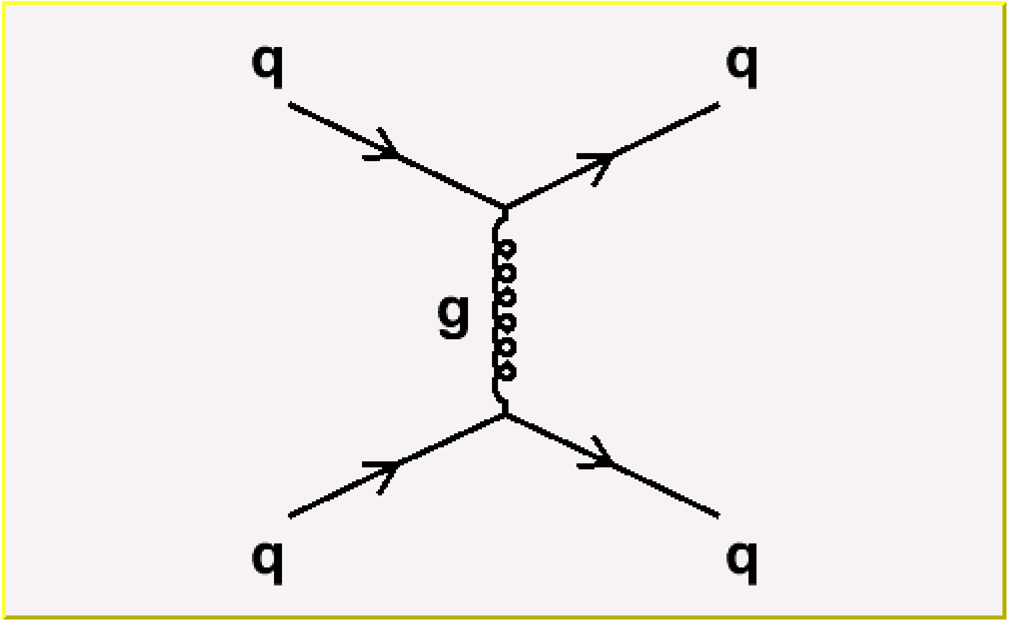
Two mediators of the weak force, Z and W bosons



$$\nu_e + e^- \rightarrow \nu_e + e^-$$

Feynman diagrams

Strong interactions

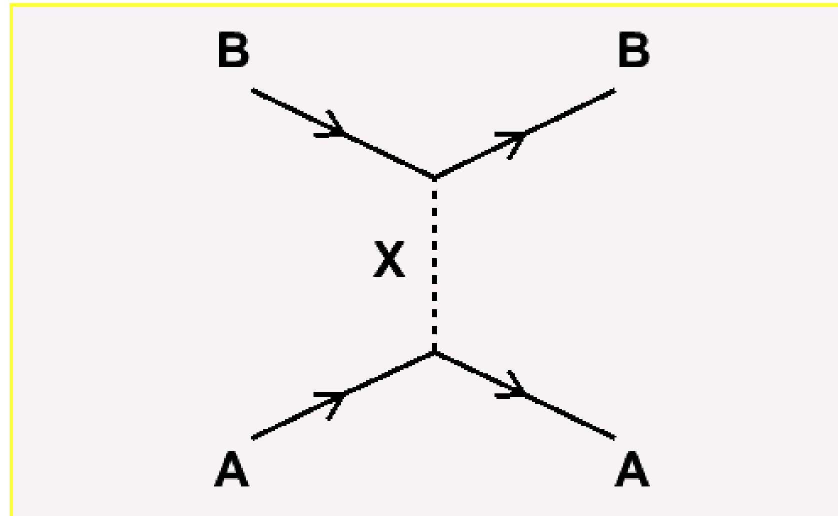


$$q + q \rightarrow q + q$$

Four-vectors

- Fundamental laws can be written in the same form for all **<u>Lorentz frames</u>**.
- Lorentz transformation relates the coordinates in two frames.
- $(ct, \boldsymbol{x}) \equiv (x^0, x^1, x^2, x^3) \equiv x^\mu$ – 4-vector, $c^2t^2 - \vec{x}^2$ - invariant
  - Convention: $\boldsymbol{x}$ or $\vec{x}$ – 3-vector
- $A \cdot B = A^0 B^0 - \vec{A} \cdot \vec{B}$
- In special relativity $P \equiv (E, \vec{p}c) \equiv (p^0, p^1, p^2, p^3) \equiv p^\mu$
  - The basic invariant $E^2 - \vec{p}^2 c^2 = m^2 c^4$

Particle exchange – range of forces



4-vector $P_A = (E_A, \mathbf{p_A}c)$

$A(M_Ac^2, \mathbf{0}) \rightarrow A(E_A, \mathbf{p_A}c) + X(E_X, -\mathbf{p_A}c)$

For A and B:

$P_AP_B = E_AE_B - \mathbf{p_A}\mathbf{p_B}c^2$

$P_A{}^2 = E_A{}^2 - \mathbf{p_A}{}^2c^2 = M_A{}^2c^4$

$E_A = (p^2c^2 + M_A{}^2c^4)^{1/2}$, $E_X = (p^2c^2 + M_X{}^2c^4)^{1/2}$ $(p = |\mathbf{p}|)$

X is **virtual** particle

$\Delta E = E_X + E_A - M_Ac^2 \rightarrow 2pc, \quad p \rightarrow \infty$

$\rightarrow M_Xc^2, \quad p \rightarrow 0$

Thus $\Delta E \geq M_Xc^2$ for all p.

Particle exchange – range of forces

- $\Delta E \geq M_X c^2$ for all p, i.e. energy is not conserved
- From Heisenberg uncertainty principle it is allowed for $t \approx \hbar / \Delta E$
- $r \approx R \equiv \hbar / M_X c$ – the maximum distance over which X can propagate, **the range of the interaction**
- $M_\gamma = 0 \Rightarrow$ infinite range of EM interaction
- $M_{W,Z} \approx 80$, 91 GeV/$c^2 \Rightarrow R_{W,Z} \approx 2\times10^{-18}$ m

Example problem

- Find the range (in fm) of the force transmitted by the exchange of a W-boson of mass 80 GeV/c^2
- Solution:

The range is

$$R \equiv \frac{\hbar}{mc} = \frac{\hbar c}{mc^2} \qquad \text{and} \qquad \hbar c = 0.2\, GeV \cdot fm$$

$$R_W = \frac{0.2}{80} fm = 2.5 \times 10^{-3} fm = 2.5 \times 10^{-18}\, m$$

Klein-Gordon equation

Merging Special Relativity and Quantum Mechanics:

from $E^2 = p^2c^2 + M^2c^4$ using $p = -i\hbar\frac{\partial}{\partial x}$ and $E = i\hbar\frac{\partial}{\partial t}$

Special Relativity + Quantum Mechanics

$$-\hbar^2\frac{\partial^2\phi(x,t)}{\partial t^2} = -\hbar^2c^2\nabla^2\phi(x,t) + M_X^2c^4\phi(x,t)$$

Describes interactions of spin 0 particles

The static form (no time dependence): $\nabla^2\phi(x) = \frac{M_X^2c^2}{\hbar^2}\phi(x)$

Yukawa potential

The Solution to the Klein-Gordon Equation:

For $M_X = 0$ (photon) the above equation is the same as for electrostatic potential. Coulomb potential:

$$V(r) = -e\phi(r) = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}$$

If $M_X \neq 0$ $\quad V(r) = -\frac{g^2}{4\pi}\frac{e^{-r/R}}{r}$

Yukawa potential

R – range,

g – **coupling constant** associated with each vertex of a Feynman diagram

Convention: $\alpha_X = \frac{g^2}{4\pi\hbar c}$ Interaction strength at r ≤ R

$$\alpha \equiv \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}$$

fine structure constant (EM interactions)

The scattering amplitude

In Feynman diagrams each vertex is associated with invariant amplitude f

$|f|^2$ – directly related to the probability of the process

Assume $g \ll \sqrt{4\pi\hbar c}$ Then perturbation theory can be used:

$$f(\vec{q}) = \int d^3\vec{x}\, V(\vec{x})\, e^{\frac{i\vec{q}\,\vec{x}}{\hbar}} \qquad \vec{q} = \vec{q}_i - \vec{q}_f$$

The integration may be done with substitutions:

$$\vec{q}\,\vec{x} = |\vec{q}|\, r\cos\theta \qquad \text{and} \qquad d^3\vec{x} = r^2 \sin\theta\, d\theta\, dr\, d\phi, \quad r = |\vec{x}|$$

For Yukawa potential:

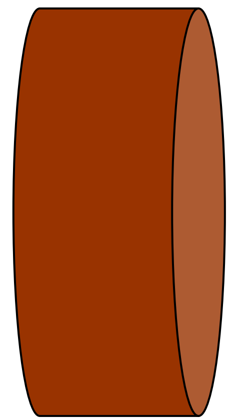
$$f(\vec{q}) = \frac{-g^2\hbar^2}{|\vec{q}|^2 + M_X^2 c^2}$$

Exchange of single particle ~ α_X^2, two particle exchange ~ α_X^4,
Example: EM interactions $\alpha \approx 1/137$ $\Rightarrow$ very small contribution from 2-particle exchange

Cross-sections – a measure of interaction probability

Beam (e-, p, π, etc…)

$J = n_b v_i$ – flux



N – number of particles illuminated by beam

Target

The rate $W_r = JN\sigma_r$

$L \equiv JN$ – contains all dependencies on densities and geometries of the beam and target – **luminosity**

Cross-section meaning:

Rate per target particle $J\sigma_r$ = Rate at which beam particles would hit a surface of area σ

dimension of area, cm^2

σ is a Lorentz invariant

Cross-section

$$\sigma \equiv \sum_r \sigma_r$$

total — partial

$$dW_r \equiv JN \frac{d\sigma_r(\theta,\phi)}{d\Omega} d\Omega$$

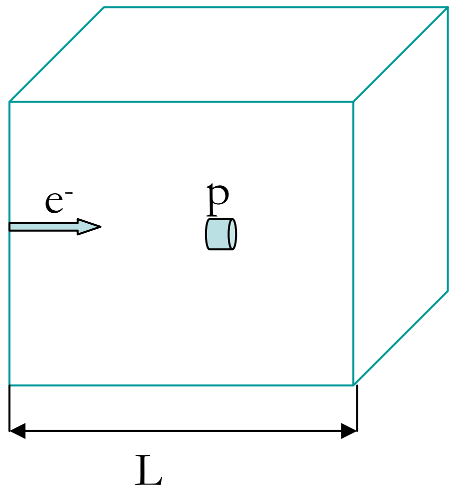
Interaction rate — differential

$$d\Omega = d\cos\theta d\phi$$

Then, partial cross-section can be obtained:

$$\sigma_r = \int_0^{2\pi} d\phi \int_{-1}^{1} d\cos\theta \frac{d\sigma_r(\theta,\phi)}{d\Omega}$$

Rate and scattering amplitude *f(q)*



$$J = n_b v_i = \frac{v_i}{V}$$

$$dW_r = \frac{v_i}{V} \frac{d\sigma_r(\theta,\phi)}{d\Omega} d\Omega$$

From perturbation theory

$$dW_r = \frac{2\pi}{\hbar} \left| \int d^3x \psi_f^* V(x) \psi_i \right|^2 \rho(E_f)$$

Fermi Second Golden Rule

Density of states

$$\psi_i = \frac{1}{\sqrt{V}} e^{\frac{i\vec{q}_i \vec{x}}{\hbar}}, \qquad \psi_f = \frac{1}{\sqrt{V}} e^{\frac{i\vec{q}_f \vec{x}}{\hbar}}$$

$$dW_r = \frac{2\pi}{\hbar V^2} \left| f(\vec{q}) \right|^2 \rho(E_f)$$

Rate and scattering amplitude (ctd)

- $\rho(E_f)$ is calculated by setting $\rho(E)dE$ equal to the number of possible quantum states of the final states particles with a total energy between *E* and *E+dE*
- It is found by firstly evaluating $\rho(q)$ and then changing variables using $\rho(q)\frac{dq}{qE}dE = \rho(E)dE$
- The possible values of *q* are restricted by the boundary conditions to be

$$q_x = \left(\frac{2\pi\hbar}{L}\right)n_x, \quad q_y = \left(\frac{2\pi\hbar}{L}\right)n_y, \quad q_z = \left(\frac{2\pi\hbar}{L}\right)n_z$$

- The number of final states with momentum lying in the momentum space volume $d^3\vec{q} = q^2 dq d\Omega$

$$\rho(q)dq = \left(\frac{L}{2\pi\hbar}\right)^3 d^3\vec{q} = \frac{V}{(2\pi\hbar)^3} q^2 dq d\Omega$$

Rate and scattering amplitude (ctd)

$$\frac{dq}{dE} = \frac{1}{v} \;\rightarrow\; \rho(E_f) = \frac{V}{(2\pi\hbar)^3} \frac{q_f^2}{v_f} d\Omega$$

⇓

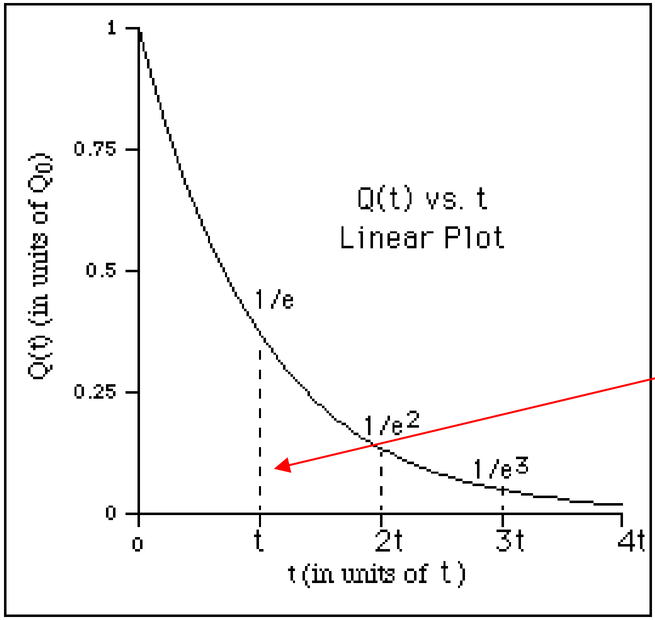
$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2\hbar^4} \frac{q_f^2}{v_i v_f} \left| f(\vec{q}) \right|^2$$

Valid for general two-body relativistic scattering process

$$A(\vec{q}_i) + B(-\vec{q}_i) \rightarrow A(\vec{q}_f) + B(-\vec{q}_f)$$

But neglects the spins of the particles

Unstable particles



- For unstable stated we measure lifetime τ or

$$\Gamma = \frac{\hbar}{\tau}$$

$$\Gamma = \sum_f \Gamma_f$$

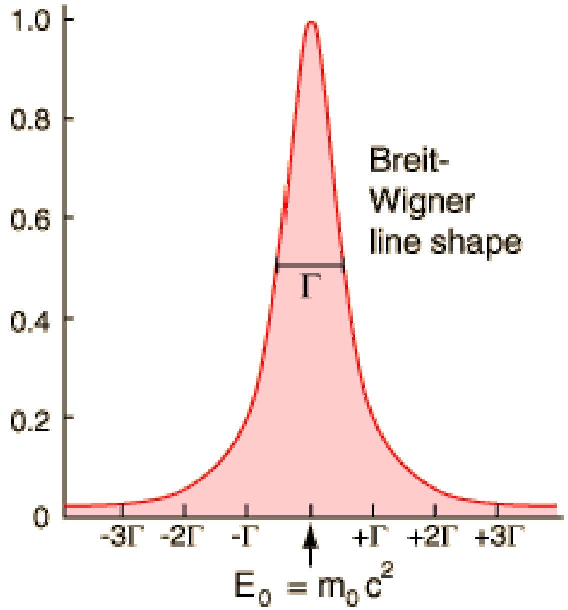
$$B_f \equiv \frac{\Gamma_f}{\Gamma}$$

Total decay width

Partial width

Branching ratio

Unstable particles.
Breit-Wigner formula



$$P_f(W) \sim \frac{\Gamma_f}{(W-M)^2 c^4 + \Gamma^2/4}$$

- M – mass of the decaying state
- W – invariant mass of the decay products

If an unstable state produced in a scattering reaction, σ for that reaction will show an enhancement. In this case we are producing a **resonance state**

$$\sigma_{if} \sim \frac{\Gamma_i \Gamma_f}{(E - Mc^2)^2 + \Gamma^2/4}$$