PC5201 Advanced Quantum Mechanics

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General Introduction

All the applications of quantum mechanics that you would have been taught up to now is based on the Schrodinger equation. This equation is deduced from the Hamiltonian formalism of classical mechanics, and so has all the invariance properties of the Hamiltonian. In particular, it is invariant under Galilean transformations. But the Schrodinger equation is not invariant under Lorentz transformations, as required by the principle of relativity. Thus, this theory will correctly describe phenomena only when the velocities of the particles involved are small compared to that of light: $v \ll c$. For example, all phenomena concerning the interaction of light and matter, such as the emission, absorption or scattering of photons, is outside the framework of non-relativistic quantum mechanics.

At first sight, one might expect that the change to a relativistic theory is possible by a direct generalisation of the formalism of non-relativistic quantum mechanics. But further consideration shows that a logically complete relativistic theory cannot be constructed without invoking new physical principles. One of the main difficulties comes from the fact that the law of conservation of the number of particles ceases to be true in general. This is due to the equivalence between mass and energy:

$$E = mc^2,$$

one of the most important consequences of the theory of relativity. Together with the Heisenberg uncertainty principle,

$$\triangle E \, \triangle t \sim \hbar \,,$$

this implies that energy fluctuations in the course of a sufficiently short observation can appear as additional matter. Hence, one needs a formalism where the number of particles is not fixed. The concept of a single-particle wave function has to be replaced by the concept of a quantum field varying over space-time. This is why the name 'quantum field theory' is usually given to relativistic quantum mechanics. Although this theory is not quite logically complete, there is very good experimental evidence for it, particularly for quantum electrodynamics (QED).

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