# 3C24 : Lecture Notes Particle and Nuclear Physics

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## Contents

# 20 LECTURE 20

## 20.1 Terminology of Nuclear Physics

Protons and neutrons bind together to make nuclei via the strong force. Protons and neutrons are collectively called nucleons. The notation is usually

- Z - Atomic Number - number of protons
- A - Mass Number - number of protons + neutrons
- N - Neutron number - number of neutrons

The notation for a nuclide is ${}^{A}_{Z}$X or just ${}^{A}$X. The charge on the atom is +Ze where e is the absolute value of the charge on the electron.

Nuclides of a given chemical can occur with different masses: these are called **isotopes** which have Z fixed but different A. All chemical properties are derived from the electrons of an atom and are therefore electromagnetic in nature. Because atoms are neutral, the number of protons defines the chemical name of a nuclide.

Nuclides of the same mass number but different Z and N are known as **isobars**. An example of two isobars is ${}^{3}$H and ${}^{3}$He.

The simplest atom is Hydrogen and is made up of 1 proton and 1 electron. This is a purely electromagnetic entity: the Electromagnetic force binds the electron to the proton and so nothing will be learned about nuclear physics from a study of Hydrogen. However, there is something interesting about the Hydrogen atom. The mass of the Hydrogen atom is slightly less than the sum of the electron and proton masses. This so called **binding energy** is only 13.6eV, but it is striking evidence to support Einstein's theory that matter and energy are equivalent.

It is usually the *atomic* weight which is measured and used for the mass number. This mass includes the mass of the electrons. This atomic mass is usually measured using atomic ions: atoms with one or more electrons stripped off which will then bend in the magnetic field of a mass spectrometer. The difference between the atomic weight and nuclear weight is very small: $E_{electronic} \approx 20.8Z^{7/3}eV$.

## 20.2 The Nucleons

The proton(p) and the neutron(n) make up the set of nucleons.

$$M_p = 938.28MeV(/c^2)$$
$$M_n = 939.57MeV(/c^2)$$



Usually in nuclear physics terminology masses are given in units of $MeV/c^2$ but we will respect natural units as before and allow $c = 1$. The first thing to notice is that the neutron is the heaviest of the two nucleons. The charge on the proton is exactly equal an opposite to the charge on an electron. The underlying symmetry which ensures that the charge of the three quarks making up a proton be exactly equal to the electron charge is as yet undiscovered and why this is so is one of the fundamental questions still left open in particle physics. The charge distribution on the proton is not point-like (neither of course is the proton itself point-like) but distributed around the center of the proton out to a radius of about 0.8fm. For the neutron, even though it is a charge zero object, it is made up of three charged quarks and its charge distribution is measured to have the positive charge concentrated in the center with the negative charge around the edge.

Both p and n have magnetic dipole moments:

$$\mu_p = 2.79284(e\bar{h}/2m_p)$$
$$\mu_n = -1.91304(e\bar{h}/2m_p)$$

and the measurement of these was clear evidence that the nucleons were not fundamental.

The nucleons are ground states of composite systems. The photon energies which are typical of *nuclear* transitions between any of these excited states are 100s of MeV to be compared with KeV and below for characteristic energies of atomic transitions.

The electrical energy associated with the charge distributions of p and n are:

$$\approx \frac{e^2}{4\pi\epsilon_0 R_p} \approx 2MeV$$

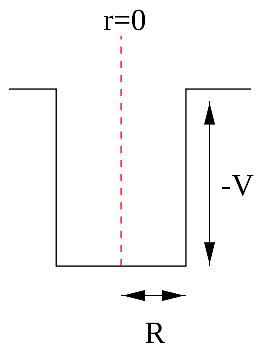
## 20.3 The Deuteron

The Deuteron is the lightest nuclear entity. This consists of a proton and a neutron. The neutron is not charged and so it is bound together with the proton solely by the nuclear force. The Deuteron also has a characteristic binding energy, but this time it is much bigger than in the case of the similar electromagnetic object (hydrogen atom).

$$m_D = m_p + m_n - 2.2\text{MeV} \quad (1)$$

indicating that the Deuteron is more tightly bound than the proton and the electron of the hydrogen atom. However, on the nuclear scale, this binding energy is rather small. The Deuteron provides us with a particularly good environment in which to do a calculation, it being very simple in terms of the forces in play. We can derive the wave-function of the Deuteron starting from energy conservation and the simple picture of a potential square well around the proton. The square well is not really exactly the correct shape of the potential (it has curved corners rather than square ones), but it does give a very good approximation to the problem.

$$V(r) = -V; r < R \quad (2)$$
$$V(r) = 0; r > R \quad (3)$$




For the ground state (triplet state),

$$V(r) = -25.5MeV; R = 2.04 \tag{4}$$

So, lets start out from energy conservation:

$$\left[\frac{p_n^2}{2m_n} + \frac{p_p^2}{2m_p} + V(r)\right]\psi = E\psi \tag{5}$$

In the center of momentum, $p_n = -p_p$, the reduced mass is given by $\mu = \frac{m_p m_n}{m_n + m_p}$ and so

$$\left[\frac{p^2}{2\mu} + V(r)\right]\psi = E\psi \tag{6}$$

from this it can be shown that if $\phi = \frac{u}{r}$

$$-\frac{\hbar^2}{2\mu}\frac{\partial^2 u}{\partial r^2} + (V - E)u = 0 \tag{7}$$

E is the binding energy of a bound state and so E=-|E|. It can be shown that if

$$u = A\sin kr; r < R \tag{8}$$
$$u = Be^{-\gamma(r-R)}; r > R \tag{9}$$

then

$$k = \sqrt{\frac{2\mu}{\hbar}(V - |E|)} \tag{10}$$
$$\gamma = \sqrt{\frac{2\mu}{\hbar}|E|} \tag{11}$$
$$k = \gamma\frac{\sin kR}{\cos kR} \tag{12}$$

You get A by normalizing

$$\int_{r=0}^{r=+\infty} \psi^*(r)\psi(R)r^2 dr = 1 \tag{13}$$



## 20.4 The Nuclear Force

The simple square well picture works well for describing the bound deuteron system as shown above. However, the nuclear force is rather complicated (not at all simple like the gauge forces) and one reason for that is that there are a multitude of particles which act as the exchange particles for the nuclear force. We mentioned earlier that pions acted as the exchange particle for the nuclear force outside of the baryon diameter of about 1fm but there are several particles such as $\rho(770)^{\pm,0}$,$\omega(782)$ which have spin 1 and larger mass than the pion which can mediate the nuclear force at smaller distances. Different masses lead to different ranges of force and different spin leads to different behaviour of the force. The nuclear force is complex : ultimately one would like to describe it on a more fundamental level e.g. by the exchange of gluons between the constituent quarks. But at present, the most successful theories are so-called effective theories where the nuclear force is decomposed into several contributions. The nuclear force can be decomposed into a central force (one that depends only on r, like the Coulomb force) and a tensor, non-central, force that depends on the relative orientation between the nucleons spin and r. Both these components further depend on the nucleon spin and orbital angular momentum.

The nuclear spin plays an important role in the nuclear force. The nuclear force is very different for singlet (S=0) and triplet (S=1) states. Indeed for deuterium the singlet state does not exist since in this case the spin part of the force is not attractive enough to overcome the repulsive components of the force. The nuclear force also has a large spin-orbit term - it is this term which is responsible for large energy splits in states that would otherwise be degenerate and explains the shell structure of nuclear levels. The strong dependence of the nuclear force on the spin and spin-orbit interactions is in marked contrast to the atomic electromagnetic force, where the spin and spin-orbit effects are small and result only in fine-structure. In the nucleus such effects are responsible for the gross structure.

There are no bound n-n or p-p states because of the shallowness of the S=0 central potential. The Pauli exclusion principle prevents a neutron and a proton from being in the same state and so if they are to be bound in the ground state they would have to have their spins aligned anti-parallel.

The complexity of the nuclear force : non-tensor nature and strong dependence on spin mean it is very difficult to do calculations for nuclei heavier than the Deuteron.

# 21 LECTURE 21

## 21.1 Heavier Nuclei

To bring two protons together, it is necessary for them to overcome the Coulomb repulsion between them. Effectively, the attractive nuclear force must overcome the potential barrier produced by this Coulomb repulsion. An extremely stable 'bundle' of nuclear matter is that containing two neutrons and two protons. This was called the alpha particle, due to its stability and ability to exist as if it were a particle in its own right.

The alpha particle has a binding energy of 28.3 MeV which is very large considering the total energy of the bundle as will be seen in the next section. It is necessary to have at least two neutrons present in order to make the strong attractive force large enough to bring two protons together. As will be seen later, nuclei always have the same number of, or more, neutrons than protons.

## 21.2 The Measurables

The strength of the binding of nuclei is an important **measurable** which we can use to try to understand more thoroughly what is going on. Figure 1(left) shows the binding energy per nucleon as a function of atomic number. The dots represent some actual data whereas the line is an approximation to the data. Only the gross features of this plot should be considered at this stage. The binding energy per nucleon peaks at about mass number 60 at a B/A of about 8.7 MeV and slowly falls off to between 7 and 8 MeV per nucleon. Another empirical piece of information is found in the observation of *which* stable nuclei exist. There is a steady increase in the number of neutrons relative to the number of protons as A increases.

Another experimental measurement that can be made is the study of particle scattering off nuclei. From *charged particle* scattering off nuclei one can measure the distribution of charge in the nucleus. A schematic diagram of the charge radius of a nucleus is shown in Figure 1(right). It has a slightly non-uniform shape due to the fact that the charge is not evenly distributed inside the nucleus but tends to cluster around the edges in order to minimize the Coulomb repulsion. It is somewhat difficult to go backwards from scattering data to the charge density shape $\rho_{ch}(r)$. However, it is possible to assume a plausible shape for the $\rho_{ch}(r)$ described by a simple mathematical expression:

$$\rho_{ch}(r) = \frac{\rho(0)}{1 + e^{(r-R)/a}} \tag{14}$$

From *neutron scattering* off different nuclei one can ascertain not only the size of the nucleus but from studying many different nuclei, the density of nuclear matter can be determined. The size of a nucleus is measured to be

$$R = r_0 A^{\frac{1}{3}} \tag{15}$$

where $r_0 \approx 1.5\ 10^{-15}$. This reflects the fact that nuclear density is found to be constant. This is depicted in Figure 2 where the matter distribution for three nuclei is shown. Inside the nucleus, the density is very similar no matter what the mass of the nucleus. This behaviour is very similar to that of a liquid and this fact did not go unnoticed. The surface of the nucleus is a well defined quantity for nuclei with A>20, but is not really applicable to light nuclei. The shape is probably spherical, but observation of electric quadrupole moments show some deviation from sphericity. For more information about the experiments undertaken, see Cottingham or Williams.



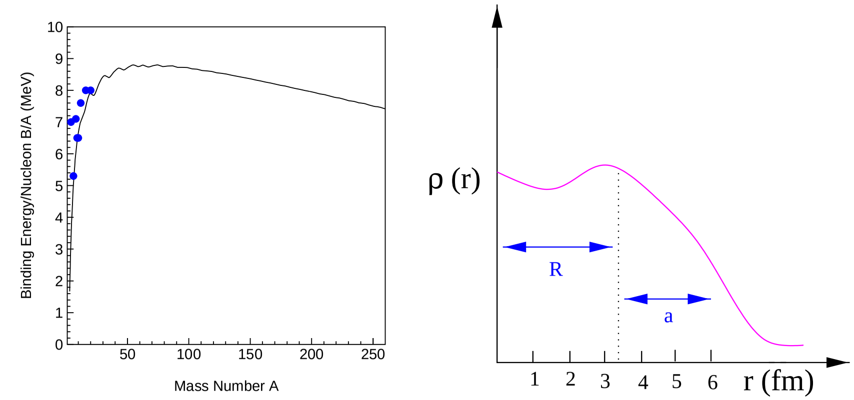



Figure 1: Left:Binding energy per nucleon as a function of the mass number A. Right: Charge density of the nucleus (schematic)

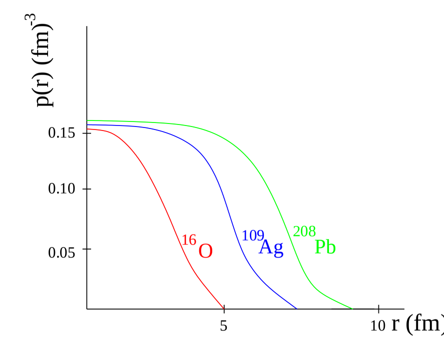



Figure 2: Nuclear density for three different nuclei.

### 21.3 Liquid drop model

Calculations for heavier nuclei are too hard to do. However, a model called the **liquid drop model** is invoked which turns out to be a very successful picture of the nucleus. In order to bring two protons together, at least two neutrons are needed to overcome the Coulomb (electromagnetic) repulsion. The forces at work inside nuclei are very strong and not necessarily simple. However, a simple consideration of the strong (attractive) force overcoming the electromagnetic (repulsive) force does give an intuitive picture. The liquid drop model, as its name suggests, treats the nucleons as if they were molecules in a liquid. Nuclear matter is pretty much incompressible, like a liquid, and the forces at work inside a liquid are also strong and complicated.

## 22 LECTURE 22

### 22.1 The Semi Empirical Mass Formula of Weizsaecker

The liquid drop model can be described mathematically by the Semi Empirical Mass Formula (SEMF). It is called *Semi* Empirical because although the constants used to describe nuclei in this way are derived from the data, the dependence of these constants has an intuitive root as we will see now. The SEMF predicts the binding energy of nuclei which has several components.

### 22.2 The Volume Term

The strength of the binding of a nucleus is proportional to the number of nucleons inside. Although the binding energy *per nucleon* falls slightly with increasing A above A≈60, the overall binding energy is increased. The contribution from this effect to the binding energy is given by

$$-B_{volume} = -u_v A \tag{16}$$

This term is a direct consequence of the short-range nature of the nuclear force. Each nucleon only interacts with a fixed number of nucleons within a certain range - it does not interact with all nucleons if it did the total binding energy would vary as A(A-1).

### 22.3 The Surface Term

A liquid drop, in the absence of external fields, will adjust its shape to minimize its energy which leads to a spherical shape in order to minimize the surface tension. Nuclei are thought to be approximately spherical, but observation of electric dipole and quadrupole moments show some deviation from sphericity. The concept of the nuclear surface is not really applicable to nuclei with A<20.

Nucleons at the surface of the nucleus will feel less attraction than those in the middle and therefore the total potential energy is lowered compared to what you would expect



if you just considered a sphere of condensed nuclear matter (i.e. this will decrease the binding energy or increase the energy needed to keep the nucleus bound). This will be most important for light nuclei and the magnitude of the effect will be proportional to the surface area.

$$-B_{surface} = u_s A^{\frac{2}{3}} \tag{17}$$

### 22.4 The Coulomb Term

As nucleons are brought together, they must overcome the Coulomb repulsion between the protons. Once the separation of the protons is about 1fm, the repulsion is over compensated. This term will decrease the binding energy (increase the energy necessary to keep all the nucleons bound).

$$-B_{Coulomb} = \frac{Z(Z-1)e^2}{A^{\frac{1}{3}}} \tag{18}$$

$$\approx \frac{(Ze)^2}{r} \tag{19}$$

For large Z it is useful to use the approximation $Z^2 \approx Z(Z-1)$

The two remaining terms in the SEMF have no basis in the gross structure of the nuclei e.g volume, surface area or total charge; but have their origins in the shell structure of the nucleus. Their origin lies in the Pauli exclusion principle and the spin dependence of the nuclear force.

### 22.5 The Asymmetry Term

The strong force is an exchange force. Exchange forces are **attractive** if the wave function is symmetric with respect to space exchange and are **repulsive** if the wave-function is anti-symmetric. Therefore, anti-symmetric pairs do not contribute to the energy because they have the wrong sign for binding. This is the reason why most stable nuclei are the ones with fewest anti-symmetric pairs. Look at two cases for the A=16 isobar as depicted in Figure 3. The lowest energy system will be the $^{16}O$ which is completely symmetric between n and p. The higher energy system will be $^{16}N$ which has an extra neutron. Because of the Pauli exclusion principle, the extra neutron must inhabit the next energy level. This leads to a term in the binding energy expression:

$$-B_{asymmetry} = u_a(N-Z)^2 A^{-1} \tag{20}$$

$$= u_a T^2 A^{-1} \tag{21}$$

The SEMF can thus far be written:

$$U(A,Z) = Zm_p + Nm_n - B_{volume} - B_{surface} - B_{Coulomb} - B_{Asymmetry} \tag{22}$$

$$= Zm_p + Nm_n - u_v A + u_c Z^2 A^{-\frac{1}{3}} + u_s A^{\frac{2}{3}} + u_a T^2 A^{-1} \tag{23}$$

where U represents the total energy of the nucleus including its rest mass.

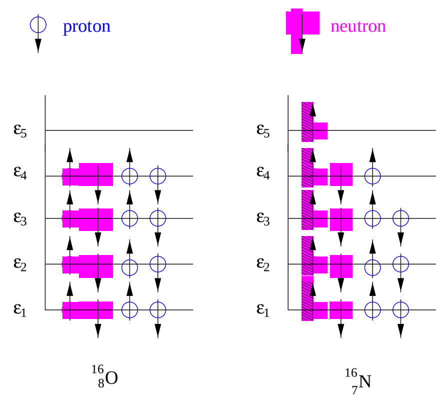



Figure 3: Left: Energy levels for $^{16}$O. Right: Energy levels for $^{16}$N

## 22.6 The Pairing Term

Following on from what we have learned about the symmetry of the exchange force, a plot of U(A,T) versus T for isobaric nuclei is a parabola with respect to T as shown in Figure 4(left), the minimum of the parabola being at $T_{min}$.

The SEMF so far excludes the possibility of more than one stable isobar. Experimentally, there is only one stable isobar for odd mass numbers but there are two and sometimes three for even mass numbers. These stable nuclei are always of the even-even variety with the odd-odd ones in between being beta-unstable in both directions (see Section 23.3). This is indicated in Figure 4(right) where U as a function of T is given for FIXED A. In this case, the difference between even-even and odd-odd nuclei is given purely by

$$\left(\frac{u_a}{A} + u_c A^{-1/3}\right) \tag{24}$$

The bulk terms being the same for a given A. The final term in the SEMF should be one depending on the type of the nucleus. The term added is

$$\pm\frac{\delta}{2A} \tag{25}$$

with the minus sign for even-even nuclei and the plus sign for odd-odd. This reflects what we learned earlier about the Deuteron: n-p pairs can be bound together because they can be in an S=1 state (their spins aligned parallel to each other) whereas n-n and p-p pairs to not add to the binding energy because the S=0 potential is not enough to bind the nucleon pairs. If there are an even number of protons and neutrons, this implies that there is maximal pairing between n and p nucleons whereas with odd-odd nuclei, it is at least guaranteed that there be an odd n and an odd p which are not paired. So even-even nuclei will have a larger binding energy than odd-odd nuclei given a fixed A.



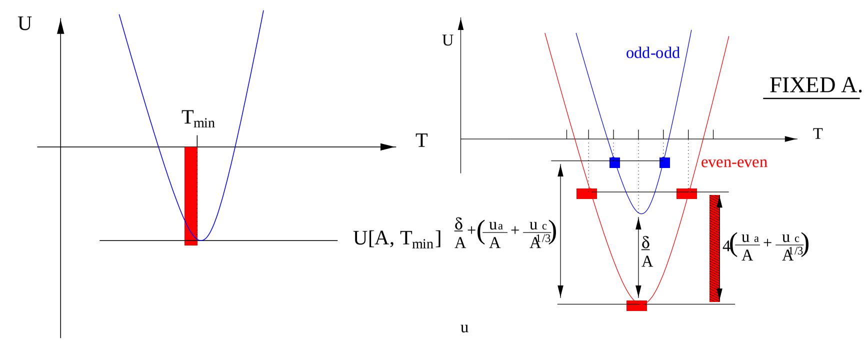



Figure 4: Left: U as a function of T from the SEMF. Right: U as a function of T for odd-odd and even-even nuclei for fixed A. The figure shows the case most favourable to the occurrence of three stable isobars (all of them even-even), with the spacing parameter $\delta$ chosen so large as to make the three even-even isobars stable.

$\delta = 0$ for odd A. The proportionality to A$^{-1}$ is assumed, since this is a symmetry effect. $\delta \approx 270$MeV for medium weight nuclei. Several texts quote the pairing term as

$$\pm\frac{\delta}{A^{1/2}} \tag{26}$$

with $\delta$=11.2MeV but either will give a sufficiently accurate determination of the binding energy. This demonstrates the purely empirical nature of this term.

The quoted values of these parameters are the ones which give the best agreement between the theory and the measured data for some particular method of determination. There are many such sets of these parameters, determined in different ways. In the literature, you will find them to vary from text to text. For example, in Williams:

$$u_v = 15.56\text{MeV}$$
$$u_s = 17.23\text{MeV}$$
$$u_c = 0.70\text{MeV}$$
$$u_a = 23.28\text{MeV}$$
$$u_p = 12.00\text{MeV}$$

The relative contributions of all the terms to the binding energy of nuclei as a function of A is shown in Figure 5.

## 22.7 Stability conditions of nuclei

There are two main effects which determine the stability of a particular nucleus and a third effect which happens rarely:

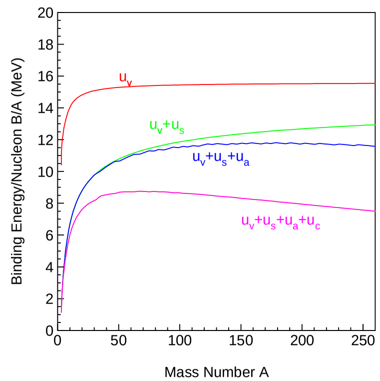



Figure 5: The contributions to the binding energy per nucleon from the volume, surface, coulomb(charge) and asymmetry terms.

- The symmetry effect: the charge independence and the exchange character of nuclear forces, together with the Pauli exclusion principle, strongly depresses the energy of nuclei with equal or nearly equal numbers of protons and neutrons
- The charge effect: the effect of the Coulomb repulsion of the protons favours nuclei with fewer protons than neutrons. The neutron proton mass difference also enters the charge effect, but only as a small correction. The charge effect increases in importance with increasing Z
- In a few special cases, the spin dependence of nuclear forces favours parallel spin over anti-parallel spin of a pair of (extra) nucleons.

# 23 LECTURE 23

## 23.1 Spontaneous Activity: Radioactivity and Fission

A group of nucleons can be bound if Z≈N. No nuclei exist which are all n or all p because of the Pauli exclusion principle and the exchange nature of the strong force. Some nuclei are unstable against a split into two or more parts. This happens if the binding energy of the parent nucleus is smaller than the sum of the binding energy of the daughter nuclei. How can these nuclei then be held together in the first place? Figure 6 shows three possibilities. At large separations the energy reaches a constant level. The energy reaches a maximum at about 1fm from the Coulomb repulsion and then at distances smaller than about 1fm, the energy drops off. In the top graph, the energy level at very close range is lower than the energy at infinite separation and so the nucleus is completely stable. In the second case, the small separation energy is higher than the large separation



energy, but in order for a spontaneous decay to occur, the decaying particle must either tunnel through this potential (Coulomb) barrier in order to reach the lower energy state or energy must be supplied to the nucleus to overcome the barrier. The first case scenario is indeed what happens in all forms of spontaneous decay. In the lower diagram, no bound nucleus is ever formed. The probability of this tunneling goes down as a function of the energy of the penetrating particle and therefore spontaneous fission is a very rare process, but spontaneous alpha decay is fairly frequent. The second case scenario, where energy is supplied to the nucleus, is what happens in induced fission.

In the case of bound nuclei, the energy difference between the small separation and the large separation is known as the separation energy. Because of the equality of neutron-neutron and proton-proton forces, the separation energy

$$S_n \approx S_p; N \approx P \tag{27}$$

This is analogous to the atomic ionisation energy which is the energy to liberate one electron; the separation energy is the energy to liberate one proton or neutron.

Nucleons are fermions and so Pauli requires that no two n or two p are in the same state. The energy levels are filled up to $E_n^F$ and $E_p^F$ (where F stands for Fermi) for neutrons and protons respectively. In $A \leq 40$, N≈Z and so $E_n^F \approx E_p^F \approx 38$MeV. In heavier nuclei, the value of $E_n^F$ can be much higher. The energy to detach a neutron is given by

$$S_n(Z,N) = B(Z,N) - B(Z,N-1) \tag{28}$$

and hence is about 8MeV. Therefore, the total depth of the neutron well is ≈46MeV. The most stable nucleus will have $E_n^F \approx E_p^F$: if these are very different the nucleus will be unstable against beta decay.

## 23.2 Radioactivity

Radioactivity is the name given to several distinctly different processes which we can now identify in terms of the fundamental forces which we have learned about. You may be familiar with the terms alpha, beta and gamma radiation, somewhat historically named.

Alpha radiation: ($\alpha$) Alpha radiation is the spontaneous emission of a helium nucleus. This is a strong interaction process.

Beta radiation: ($\beta$) Beta radiation is just the weak decay of one of the quarks in a nucleon to an electron and an electron anti-neutrino. The electron is emitted (the beta ray) while the neutron(udd) becomes a proton(uud) via a $W^-$ or under certain energetically favoured situations a proton becomes a neutron via a $W^+$. See Figure 7.

Gamma radiation: ($\gamma$) Gamma radiation is just another name for a particular energy of photon. These energies are in the MeV range and the photons come from the nucleons inside the nucleus rearranging themselves into a more energetically favourable configuration with the emission of the resultant energy given off as a photon. X rays on the other hand, come from electrons in the atoms rearranging themselves and dropping down across energy levels giving out photons. These photons are typically in the KeV range of energies.

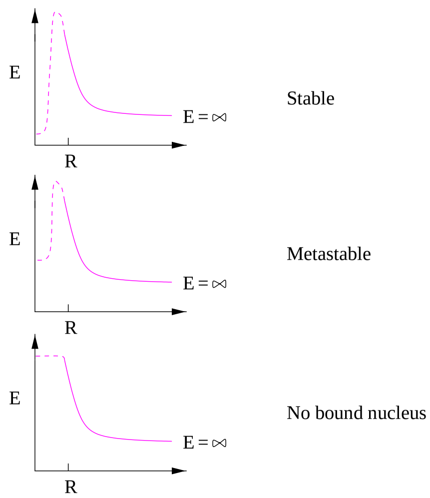



Figure 6: Three different cases for the binding of nuclei

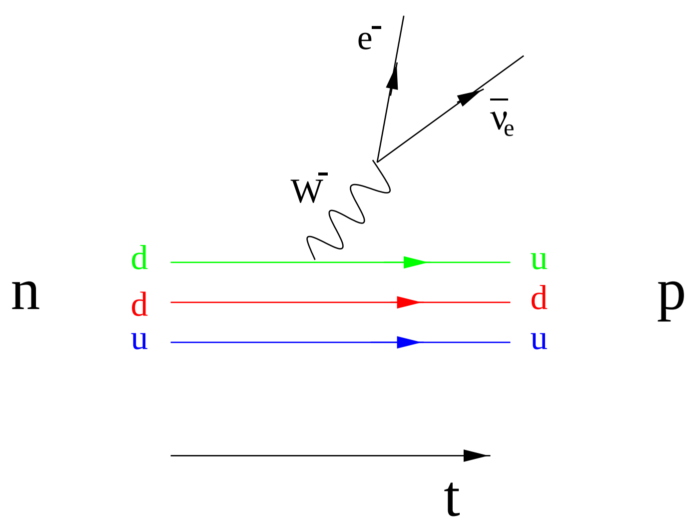



Figure 7: Feynman diagram of 'beta decay': a weak decay of a neutron to a proton, electron and anti-neutrino.



## 23.3 Energy considerations for stability against $\alpha$ and $\beta$ radioactivity

Although the liquid drop model of the nucleus has its shortcomings, the SEMF which has been derived using the model describes several gross features of nuclei such as the binding energy curve. It can be used to estimate the stability conditions of nuclei.

$\alpha$ decay occurs when the binding energy of the parent nucleus is less than the sum of the binding energies of the daughters in a decay:

$$B_1 < B_2 + B_3 \tag{29}$$

Because of the gradual fall off of B/A with increasing A, this may be true for high A nuclei into two intermediate A nuclei. For any beta stable nucleus with A>85 this is true. $\alpha$ particles are the most stable clump of nuclear matter with 2 neutrons,2 protons and a binding energy of 28.3 MeV. However this does not take into the account the rate at which the $\alpha$ decay occurs. The decays only actually proceed (despite being energetically viable) at an observable rate for A $>$ 220. This is because $\alpha$ decay proceeds via the quantum mechanical tunneling through the Coulomb barrier. The success rate of this tunneling is essentially zero until the KE of the alpha particle is $>$ 6 MeV and this only happens when A is large.

The difference in energy between the parent and daughter nuclei for $\beta$ decay can be written:

$$\begin{aligned} \Delta U_{Z\to Z+1,N\to N-1} &= U(Z,N) - U(Z+1,N-1) && (30)\\ &= B(Z+1,N-1) - B(Z,N) + (m_n - m_p) && (31) \end{aligned}$$

We can look at three different general examples:

$$\begin{aligned} {}_ZX &= {}_{Z+1}Y + e^- + \bar{\nu_e}; \Delta > 0.511\text{MeV} && (32)\\ {}_{Z+1}Y &= {}_ZX + e^+ + \nu_e; \Delta < -0.511\text{MeV} && (33)\\ {}_{Z+1}Y + e^- &= {}_ZX + \nu_e; \Delta < 0.511 - \epsilon\text{MeV} && (34) \end{aligned}$$

where $\epsilon$ is the binding energy of the electron in the state from which it is captured. Any excess of energy appears as kinetic energy of the emitted electron and neutrino. Therefore we see that a nucleus is *only* $\beta$ stable if either:

$$\begin{aligned} \Delta U_{Z\to Z+1,N\to N-1} &< 0.511\text{MeV or} && (35)\\ \Delta U_{Z\to Z-1,N\to N+1} &< -0.511\text{MeV} && (36) \end{aligned}$$

this leads to the interesting result that two *stable* isobars *must* differ by more than one unit in Z. X and Y isobars differing by one unit cannot be stable against $\beta$ decay *unless* $\Delta U = 0.511$MeV. The lifetimes for this process are on the order of a fraction of a second which is very slow. This is because it is a weak interaction and this force is very weak.

The SEMF can be used to determine whether a particular nucleus is stable against spontaneous decay. The SEMF can be written as

$$U = \Delta + \Lambda Z + \Gamma Z^2 \tag{37}$$

where

$$\Delta = Am_n - u_vA + u_sA^{\frac{2}{3}} + u_aA \pm \frac{\delta}{2A} \quad (38)$$
$$\Lambda = m_p - m_n - 4u_a \quad (39)$$
$$\Gamma = \frac{u_c}{A^{\frac{1}{3}}} + \frac{4u_a}{A} \quad (40)$$

For odd isobars, $\delta = 0$ because there is only one parabola of stable nuclei as shown in Section 22.6. Now, if the weak decay arguments above allow only one stable isotope (which has the smallest mass for a given Z) which is an even-even combination even isobar (the odd-odd combination even isobars being beta unstable) then $\frac{dE}{dZ} = 0$ at the minimum Z:

$$\left(\frac{\partial U}{\partial Z}\right) = \Lambda + 2\Gamma Z \quad (41)$$
$$= m_p - m_n - 4u_a + 2\frac{u_c}{A^{1/3}}Z + 8\frac{u_aA}{Z} \quad (42)$$
$$= 0 \quad (43)$$
$$Z = \frac{-\Lambda}{2\Gamma} \quad (44)$$
$$= \frac{A}{2}\left[\frac{m_n - m_p + 4u_a}{u_cA^{2/3} + 4u_a}\right] \quad (45)$$
$$= \frac{A}{2\left[\frac{4u_a + u_cA^{2/3}}{4u_a + m_n - m_p}\right]} \quad (46)$$

This expression can be reduced by retaining only the leading terms and neglecting the neutron-proton mass difference, to

$$Z \approx \frac{A}{2} - \frac{u_c}{8u_a}A^{\frac{5}{3}} \quad (47)$$

The ratio of $u_c$ to $u_a$ enters because the stability against $\beta$ decay is determined by the competition between the charge effect which favours large neutron excesses and the asymmetry effect which favours $T = 0$. For the same reason the dependence on the mass number is $A^{\frac{5}{3}}$: the charge effect varies like $A^{\frac{2}{3}}$ and this must be divided by the symmetry effect which varies like $A^{-1}$. The effect of this consideration is shown in Figure 8 where the stable odd-A nuclei are shown as dots on the graph of Z vs N and the line is N=Z. The value of $T_{min}$ determines the relationship between N and Z.

Another set of constants for the SEMF IS:

- $u_v = 14.1$MeV
- $u_c = 0.15$MeV
- $u_a = 18.1$MeV
- $u_s = 13.1$MeV



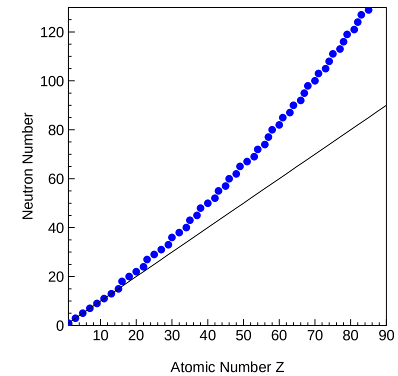



Figure 8: N vs Z for stable nuclei

If you know all the constants then equation 45 becomes:

$$Z \approx \frac{A}{2(0.98 + 0.002A^{2/3})} \quad (48)$$

which falls short of A/2 and more so as A increases as shown in the figure. The reason why there are several sets of the constants is that in order to measure them, a fit has to be done to the binding energy per nucleon curve in terms of the 5 parameters, and different approaches to doing this fit result in different values of the constants.

To look at the stability of a nucleus against $\alpha$ decay, we differentiate the SEMF with respect to A and Z.

$$\Delta U = U_{parent} - (U_{daughter} + U_\alpha) \quad (49)$$
$$= U(Z, A) - U(Z-2, A-4) - U(2, 4) \quad (50)$$
$$= \frac{\partial U}{\partial A}\Delta A + \frac{\partial U}{\partial Z}\Delta Z - U(2, 4) \quad (51)$$
$$= \frac{Z}{A^{1/3}}(\chi_1 - \chi_2\frac{Z}{A}) + \frac{\chi_3}{A^{1/3}} - \chi_4(1 - \frac{2Z}{A})^2 - 28.1MeV \quad (52)$$

where $\frac{\partial U}{\partial Z}$ is given by equation 41, $\Delta A$=4, $\Delta Z$=2 and

$$\frac{\partial U}{\partial A} = m_n - u_v + \frac{2}{3}u_sA^{-1/3} - \frac{1}{3}u_cZ^2A^{-4/3} \quad (53)$$

In order to get the expression for $\Delta U$ in (52 you need to know that the binding energy of an alpha particle is 28.3MeV and that $u_v = 14.1MeV$ and $-4u_v + 28.3$MeV = -28.1MeV.

# 24 LECTURE 24

## 24.1 Fission

We can use the SEMF to find out about the energetically allowed decays of nuclei. From the graph of the binding energy per nucleon in Figure 1(left) it is apparent that the most stable nuclei will have A≈60. Actually, $^{56}$Fe is the most stable nucleus. It is also apparent that energy can be released by reducing the mass if A> 60 (called fission) or increasing the mass if A< 60 (called fusion). If Q is the difference in the binding energy between the decay products and the parent nucleus;

$$Q(A,Z) = B(A-X, Z-Y) + B(X,Y) - B(A,Z) \tag{54}$$

then if Q is positive, the decay is energetically favourable. Following on from this equation

$$\begin{aligned} \Delta B &= u_v\left[A - X + X - A\right] & (55)\\ &+ u_s\left[A^{2/3} - X^{2/3} - (A-X)^{2/3}\right] & (56)\\ &+ u_a\left[\frac{(A-2Z)^2}{A} - \frac{(X-2Y)^2}{X} - \frac{(A-X-2Z+2Y)^2}{(A-X)}\right] & (57)\\ &+ u_c\left[\frac{Z^2}{A^{1/3}} - \frac{Y^2}{X^{1/3}} - \frac{(Z-Y)^2}{(A-X)^{1/3}}\right] & (58) \end{aligned}$$

The first term is obviously zero. This means that the attractive part of the strong force which comes from the bulk attraction, i.e. number of nucleons is not involved in the balance of energy from fission. The Coulomb interaction is obviously repulsive and energy will be released, the surface energy will always prefer a smaller "drop" of nuclear material and the asymmetry term may or may not be important depending on what the decay products are. This is summarised in Figure 5 which shows the relative contributions to the SEMF from these different terms as a function of A.

The SEMF predicts that the energy release is a maximum when the two fragments are of equal size, but there is usually a striking difference between the sizes of the products which are thought to be due to shell structure effects. For fission into two equal parts

$$\begin{aligned} Q &= -u_s A^{2/3}[2\left(\frac{1}{2}\right)^{2/3} - 1] - u_c\frac{Z^2}{A^{1/3}}\left[2(\frac{1}{2})^{5/3} - 1\right] & (59)\\ \frac{Z^2}{A} &> \frac{u_s(2-2^{2/3})}{u_c(2^{2/3}-1)} & (60)\\ \frac{Z^2}{A} &> 18 & (61) \end{aligned}$$

This condition is satisfied by $\beta$ stable nuclei heavier than $^{98}_{42}$Mo. A large amount of energy is released in symmetric fission for example for $^{235}$U, about 170MeV per fission is given out which is about $10^7$ times greater than heat given out in a combustion process. An **order of magnitude** calculation can be performed to demonstrate this fact using only electromagnetic considerations. This is a particularly simple calculation which is valid to within a factor of two (the difference between $u_v$ and $u_c$).



Consider a chemical reaction such as occurs in an explosion of TNT. A valence electron will be torn out of its shell and replaced around another atom or compound. The energy emitted (from an inverse square potential e.g. the Coulomb one) is given by

$$E = \frac{e^2}{R} \tag{62}$$

R in the case of the atomic radius is $R_{atom}$=$10^{-10}$m which is $10^4$ times the size of a Uranium nucleus. The electron can be considered as moving away to infinity from $R_{atom}$.

Uranium has 92 protons so $E_{parent} = (92)^2/R_{nucleus}$ and $R_{nucleus}$ is $10^{-14}$m. After the decay to two equal sized pieces, the total energy of the system is

$$2E_{daughter} = \frac{2.1.26.(46)^2}{R_{nucleus}}. \tag{63}$$

The factor of 1.26 comes in because the two daughters now have smaller radii than the parent by a factor $1/^3\sqrt{2}$. The difference between the original energy and the final energy is $3\cdot10^{17}$ (in some units) compared with $1\cdot10^{10}$ in the case of the chemical reaction. This shows that the nuclear reaction gives out on the order of a whopping $10^7$ times more energy than a chemical process. In the case of explosives, 1kg of $^{235}$U≡ $3\cdot10^3$TONS of TNT.

The transuranic elements can be made manifestly unstable by bombarding them with neutrons. A schematic diagram of what happens to a nucleus while undergoing fission is shown in Figure 9. If the nucleus is given a certain excitation energy, it will oscillate between an oblate and a prolate ellipsoid (between the points A and B). If enough energy is given to the nucleus to overcome the potential barrier then the nucleus will fission. The Coulomb barriers inhibiting spontaneous fission are in the range 5-6MeV for A≈240. Therefore, if a neutron with zero kinetic energy enters a nucleus, the excitation energy will be equal to the neutron binding energy. Now for $^{235}$U, a zero kinetic energy neutron will have a binding energy of 6.46MeV which is enough to push the nucleus over the Coulomb potential and fission quickly occurs (within $10^{-14}$ seconds). However, for $^{238}$U, the binding energy of the last neutron is only 4.78MeV which is not enough to cause the nucleus to fission.

The differences in the binding energy of the last neutron in even-A and odd-A nuclei are incorporated in the SEMF in the pairing term. The odd A nuclei

$$^{233}\text{U}, ^{235}\text{U}, ^{239}\text{Pu}, ^{241}\text{Pu}$$

are fissile, i.e. fission is induced by a zero kinetic energy neutron whereas the even A nuclei

$$^{232}\text{Th}, ^{238}\text{U}, ^{240}\text{Pu}, ^{242}\text{Pu}$$

need an energetic neutron to induce fission.

One decay chain of $^{235}$U is

$$^1n + ^{235}U \rightarrow ^{236}_{92}U^* \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^1n + Q$$

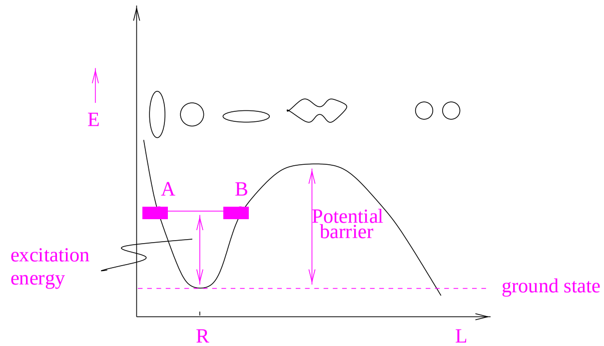



Figure 9: Energy levels and break up of a heavy nucleus

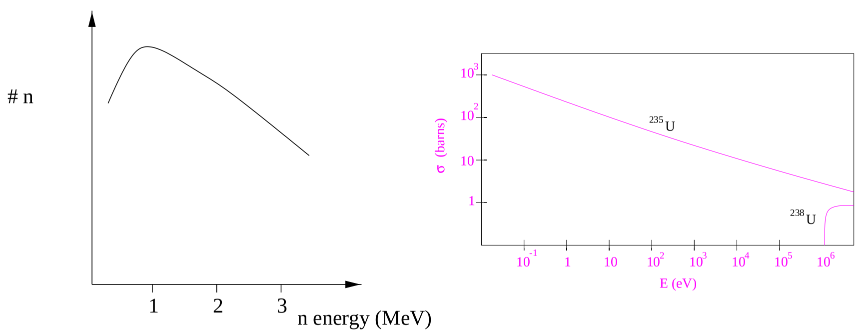



Figure 10: Left: Energy of neutrons from fission of $^{235}$U . Right: Fission and total absorption cross section for neutrons.

The importance of the neutrons in this decay chain cannot be understated. One neutron is needed to induce a fission in a nucleus but three are produced in the fission itself, leading to the possibility of inducing three more nuclei to fission. This is called a chain reaction. The neutrons have a certain absorption length which depends on their energy. The energy distribution of neutrons from $^{235}$U decay is shown in Figure 10 (left) and the mean energy is about 2 MeV. Their absorption cross section for fission is shown as a function of this energy in Figure 10 (right).

Lets make a few definitions:

ELASTIC SCATTERING
$^{23X}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{23X}\text{U} + {}^{1}_{0}\text{n}$
where 23X could be 235 or 238.

INELASTIC SCATTERING
$^{23X}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{23X}\text{U}^{*} + {}^{1}_{0}\text{n}$



where 23X could be 235 or 238

RESONANT CAPTURE
$^{23X}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{23X+1}\text{U}^{*} + \text{m}\gamma$
where 23X could be 235 or 238.

A 2 MeV neutron has a low cross section for interaction with a $^{235}$U nucleus and the probability that it will induce fission is only 18% compared with resonant capture. However, it will undergo elastic collisions with nuclei and lose energy, thereby increasing its probability of interaction. The **mean free path** of a 2MeV neutron in U (the average distance it travels before capture) is 3cm and it does this in $1.5 \cdot 10^{-9}$s. The average number of collisions is $\approx 6$, and assuming a random walk, the distance it travels from where it started is $\sqrt{6} \cdot 3\text{cm} = 7\text{cm}$ and this takes about $10^{-8}$s.

The energy release in fission can be separated into two parts: the fast component which is given out in $10^{-14}$s and the slow component which is emitted at anytime after about 13s, and is sometimes delayed by decades. This slow component comes from $\beta$ emissions and neutron emissions from the neutron-rich fission daughters. This slow component is vital in controlling nuclear fission reactors.

# 25 LECTURE 25

## 25.1 Why $^{238}$U is safe

Figure 10(right) shows the fission absorption cross section for neutrons in $^{238}$U . There is essentially zero probability that a neutron will cause fission in $^{238}$U unless its energy is above 1.4MeV which accounts for about 75% of the emitted neutrons. Of these neutrons which have enough energy, 95% of them are slowed down to below this 1.4MeV threshold by just a couple of inelastic collisions before being absorbed while 5% of them produce a fission. Therefore, there is a factor of 0.037 times the number of neutrons emitted in a fission. This average number of neutrons per fission is 2.2 (for all U chains) so the **effective** number of neutrons per fission for $^{238}$U is 2.2 times $0.04 \approx 0.09$. Not enough to produce a chain reaction. Indeed with this fission probability one would need to produce 25 neutrons per fission to ensure that at least one neutron is absorbed for fission by $^{238}$U . The fission probability in $^{235}$U is far higher ($\sim 20\%$) in the energy region of the neutrons emitted in fission and increases substantially (to  80%) in the thermal neutron energy region (E ¡ 0.1 eV = 1100 K). Thus to achieve sustainable fission in Uranium one must either **enrich** the fraction of 235 considerably (e.g. to around 20%) or else use neutrons of low energy where for 235 the fission probability is high. Both of these approaches are utilised in fission nuclear reactors.

## 25.2 Critical Mass in $^{235}$U

The fact that neutrons have a mean free path in the Uranium before they are absorbed leads to the concept of a critical mass. If the block of $^{235}$U is big enough such that every neutron will travel its 7cm and then be absorbed, the chain reaction will continue.

If, on the other hand, the block of Uranium is so small that a substantial fraction of the neutrons escape the Uranium altogether before being absorbed, then if the number of neutrons actually re-absorbed is less than 1 per fission the chain reaction will stop. Because fissile material is very expensive, it is useful to be able to minimize the size of the critical mass $m_c$ necessary to sustain a chain reaction.

In 1kg of $^{235}$U , there are about $2.5{\cdot}10^{24}$ nuclei, so it will take about 80 generations to fission the entire kg which will take 0.8$\mu$s (80 times $10^{-8}$s from the neutron path). However, as the chain reaction starts, the mass of Uranium will be heating up because each fission gives out 170 MeV. The temperature rises dramatically and the mass will expand, the density will therefore be lowered and the neutron mean free path will be increased until too many neutrons escape and the chain reaction stops. If only 1% of the available nuclei fission, the average velocity of the nucleons is $10^8$cm s$^{-1}$ and an expansion of a few cm will stop the reaction. This implies that the whole reaction must occur in $5{\cdot}10^{-8}$s. This looks like there is only time for about 5 further generations of fission after 1% of them have already fissioned.Because the last generations give out the most energy, it is not possible to slow the neutrons down because of time limitations, and so they must be used as they are.

If the probability that a newly created neutron induces fission is $q$, then each neutron leads to (on average) $\nu q - 1$ additional neutrons in time $t_p \approx 10^{-8}s$. If there are $n(t)$ neutrons present at time t then:

$$n(t+\delta t) = n(t) + (\nu q - 1)n(t)\frac{\delta t}{t_p} \tag{64}$$

and if $\delta t$ is small then

$$\frac{dn}{dt} = \frac{(\nu q - 1)}{t_p}n(t) \tag{65}$$

which has the solution

$$n(t) = n(0)e^{(\nu q - 1)t/t_p} \tag{66}$$

so the number of neutrons will increase or decrease *exponentially* depending on whether $\nu q < 1$ or $\nu q > 1$. For $^{235}$U fission, the average number of neutrons $\nu = 2.2$, so the number of neutrons will increase exponentially if

$$q > \frac{1}{\nu} \approx 0.4 \tag{67}$$

If the mass is too small, i.e. sub-critical, the reaction damps down exponentially too. The critical radius for $^{235}$U is ($\nu q = 1$) is about 8.7cm corresponding to a mass of 52Kg.

Plutonium ($^{239}$Pu ) is more efficient because there are 3 neutrons on average given out in the fission instead of 2.2 in Uranium. It is produced in the decay:

$$\begin{array}{l} ^{239}_{92}\mathrm{U} \rightarrow ^{239}_{93}\mathrm{Np} + \mathrm{e}^- + \bar{\nu_e} \\ ^{239}_{93}\mathrm{N} \rightarrow \mathrm{p}^{239}_{94}\mathrm{Pu} + \mathrm{e}^+ + \nu_e \end{array}$$

The critical mass of Plutonium is 11Kg.



## 25.3 Thermonuclear Devices

The property of fission has been utilized in producing thermonuclear devices. The critical mass is effectively reduced by using a **tamper** which surrounds the fissile material to reflect the escaping neutrons and also to retard the expansion. Critical masses for Plutonium and Uranium are shown in Table 1 with and without a tamper. For the tamper, an extremely dense material is needed such as Au, W (Tungsten), Re (Rheunium) or $^{238}$U .

| | Pu | Pu(with Tamper) | $^{235}$U | $^{235}$U (with tamper) |
|---|---|---|---|---|
| Critical Mass(kg) | 11 | 5 | 56 | 15 |

Table 1: Critical masses for fissile materials

Using fissile Plutonium in a bomb reduces the critical mass substantially over $^{235}$U , but the efficiency is still quite low because of the time considerations.

To detonate the critical mass, two sub-critical masses must be brought together very quickly along with a neutron source to produce the first neutrons for the chain reaction.

The damage caused by a nuclear explosion is terrifying. There are two effects: the shock wave which destroys everything within miles but following that is air incandescence: where the air itself is set alight. A 1kg device 0.3s after detonation at 425ft will produce a temperature of 7000C (the temperature at the sun's surface is only 5000C). 1 mile from the explosion, brightness will be 3.5 times that of the sun.

# 26 LECTURE 26

## 26.1 Measurements of $\gamma$ rays from cruise missiles

The problems of nuclear decomissioning are manyfold, but nevertheless the various treaties which are being negotiated to ensure the non-proliferation of nuclear weapons are of paramount importance.

One issue which concerns physicists is the identification of particular warheads during the process of decomissioning. All parties agree that the weapons must be destroyed but at the same time, neither 'side' wants to give away their nuclear technical secrets. As will be seen in the following, gamma ray templates of nuclear weapons can be made, and kept secret, so that the identity of a particular warhead can be verified without loss of sensitive information.

The following describes highlights of an experiment done on the Slava warship on the Black Sea which was equipped with a single nuclear warhead in the outside forward launcher on the starboard side.

A germanium detector kept at low temperature was used to measure the energy spectrum of gamma rays. The detector had energy resolution $\sigma(E)$, of 2KeV at $E_\gamma \approx 1000KeV$. Measurements were taken at four different positions

- 24 minutes on the launch tube
- 10 minutes on empty tube
- 27m from launch tube
- 32m from launch tube

Several software packages were used to identify the peaks in the spectrum. An energy calibration was done on the detector using the two peaks from $^{60}Co$. As can be seen from Figure 11, there are many lines due to $^{235}$U or $^{239}$Pu. The presence of either suggests some nuclear warhead present.

Some interesting detective work was undertaken. For example, the presence of $^{232}$U as indicated from the peaks at 583KeV and 2614.3KeV, show that the Uranium used for the missile had come from a reactor: $^{232}$U is not naturally occurring. The statistical analysis performed showed that the maximum distance that such measurements could be performed would be between 4-6m. Beyond that, the signal would not be significant above the background.

# 27 LECTURE 27

## 27.1 Reactors

Fission also provides the possibility of power generation. Figure 12 shows a schematic diagram of a **thermal reactor**. A more complete picture is shown in Figure 13. Thermal reactors have the advantage of utilizing natural Uranium because the neutrons are slowed down to **thermal** energies (<.1eV). The neutrons are slowed down by the **moderator** which is usually graphite or heavy water ($D_2O$). The moderator must have low mass number and a low neutron absorption cross section in order to maximize the energy loss of the neutron per collision while at the same time not absorbing the neutron completely.

The fuel rods are usually made of ceramic Uranium Dioxide.

The thermal neutrons must then go out and look for $^{235}$U in which to induce fission as they slow down. The probability for the neutron producing fission in $^{238}$U is very small until it has slowed down to $< 0.1eV$ kinetic energy where it is below the $^{238}$U fission threshold. In fact, if the neutron slows down in Uranium and not in a specially selected moderator, it is more likely to be resonantly captured by $^{238}$U to make $^{239}U + \gamma$ and there are no further neutrons. This is another manifestation of the fact that $^{238}$U is safe.

A second type of reactor is the **fast breeder** reactor. In this type of reactor, fission is induced directly by the fast neutrons and so either the Uranium must be enriched to 20% $^{235}$U or a fuel rod containing 20% $^{239}$Pu is used in order to sustain a constant rate of fission. This does not utilize a moderator, but relies on the fact that some fraction of the neutrons are resonantly captured by the $^{238}$U which eventually produces $^{239}$Pu after some days:



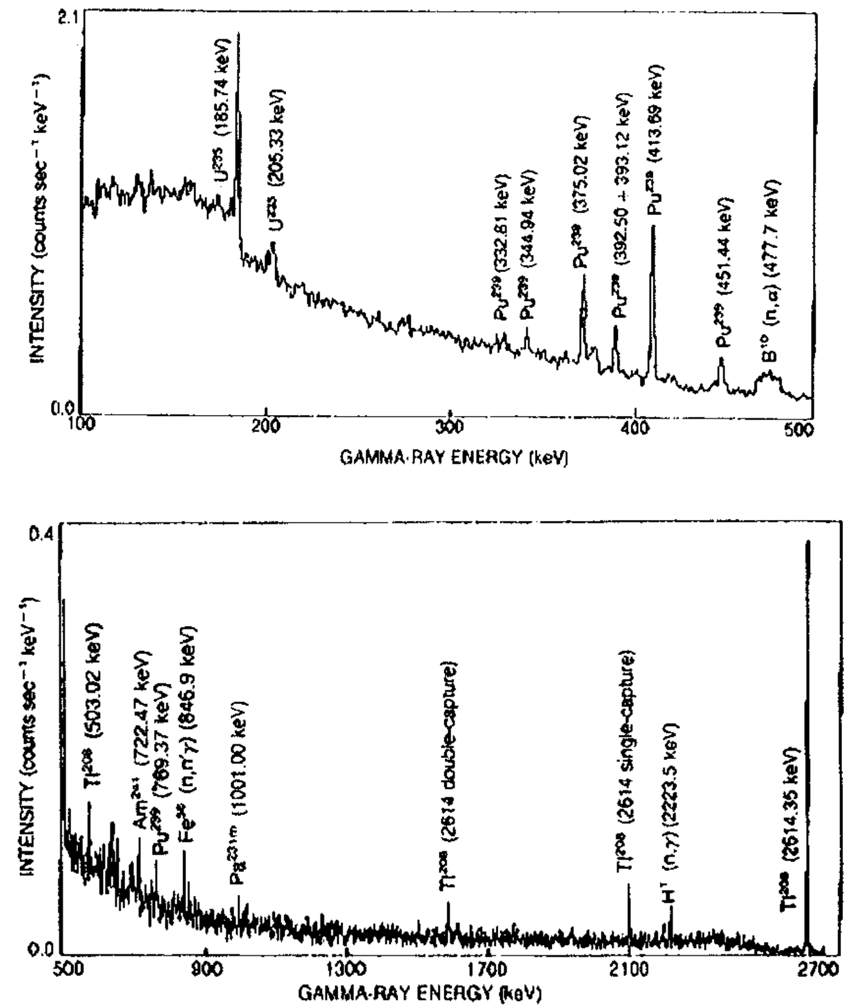



Figure 11: Energy spectrum of gamma rays measured on the launch tube

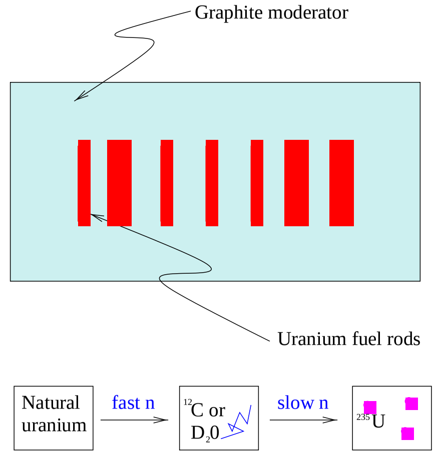



Figure 12: Schematic diagram of a thermal reactor

$$^{238}\text{U} + {}^{1}\text{n} \rightarrow {}^{239}_{92}\text{U} + \gamma$$
$$^{239}_{92}\text{U} \rightarrow {}^{239}_{93}\text{Np} + \beta^{-} + \bar{\nu}_e \text{ T=23.5min}$$
$$^{239}_{93}\text{Np} \rightarrow {}^{239}_{94}\text{Pu} + \beta^{-} + \bar{\nu}_e \text{ T=2.35 days}$$
$$^{239}_{94}\text{Pu} \rightarrow {}^{235}_{92}\text{Pu} + {}^{4}_{2}He \text{ T=24,360 years}$$

This Pu is easily extracted from the Uranium because it is a different *chemical* and so it can be chemically extracted (lets the electrons do the work). Enriching Uranium is much harder because the $^{235}$U must be extracted from $^{238}$U which is the same chemical and so chemical procedures cannot be utilized and centrifugal or mass spectrometry technology must be used.



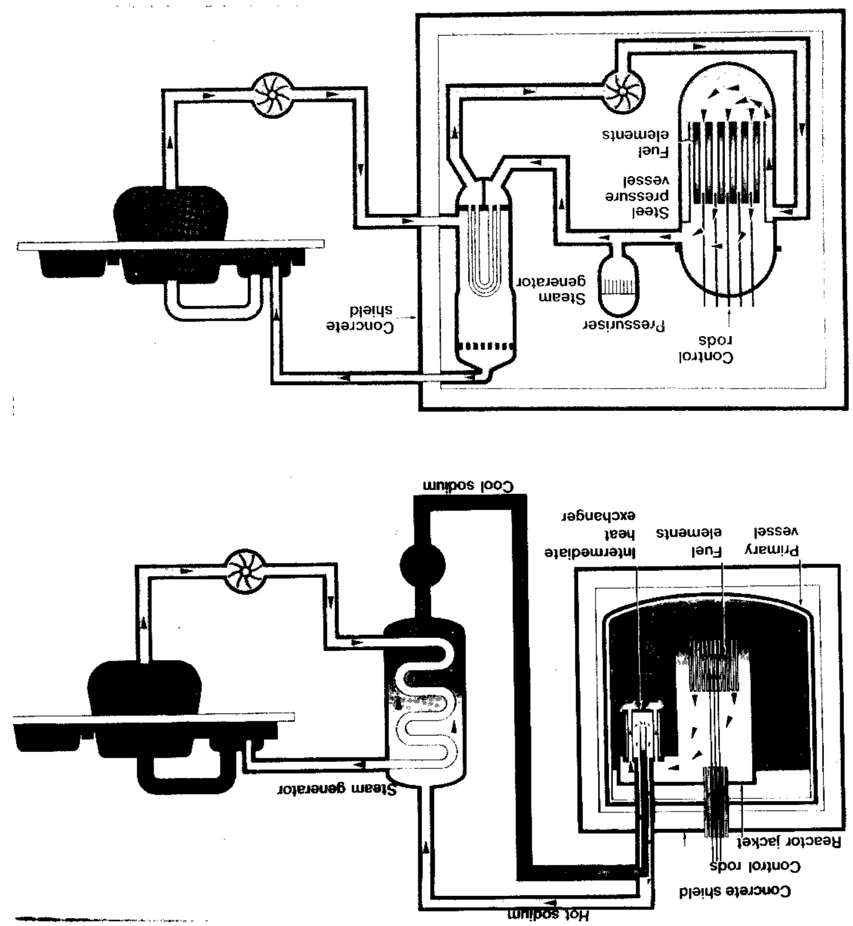



Figure 13: Thermal and Fast breeder reactors

# 28 LECTURE 28

## 28.1 Fusion in the Sun

In stars, gravity, the weak, EM and strong interactions all play a role. Gravity is a long range and universally attractive force. Any homogeneous mass of gas at low temperature will contract to form a star under gravity. The rate of collapse is determined by the extent that the build up of heat and pressure balances gravity. Some important parameters of the sun are:

- $M = 1.99\ \dot{1}0^{30}$Kg
- $R = 6.96\ \dot{1}0^{8}$m
- $\mathcal{L} = 3.86\ \dot{1}0^{26}$W

The collapse will stop when the pressure and gravity balance. This happens when the interior becomes hot enough to ignite the Hydrogen burning cycle:

$$p+p \rightarrow {}^2_1H + e^+ + \nu_e + 0.42MeV$$

This reaction occurs *very* rarely. It not only must overcome the coulomb repulsion between the two protons but it must also rely on the weak interaction (which has very slow time scales because its so weak) for the beta decay. Thanks to this fact, the sun is still burning today. This reaction is so rare in fact that it has never been observed in a laboratory environment.

## 28.2 Fusion in the Lab

Fusion is potentially a very great source of energy. The binding energy per nucleon for nuclei with masses lower than Fe is lower than that for Fe and so the possibility of producing energy by fusing nuclei together presents itself. Two possible reactions for producing fusion in the lab are:

$$ {}^2_1H + {}^2_1H \rightarrow {}^3_2He + n + 3.27\ MeV$$
$$ {}^2_1H + {}^2_1H \rightarrow {}^3_1H + n + 4.03\ MeV$$

One reason for trying these reactions is that 0.015% of all hydrogen is deuterium; for example it is abundant in sea water. In addition the isotope separation is easy because of the mass ratio of 2:1. Then, the final reaction can be attempted:

$$ {}^2_1H + {}^3_1H \rightarrow {}^4_2He + n + 17.6\ MeV$$

We can do a neat calculation. How many neutrinos reach the earth from the Sun assuming that only the PPI chain is responsible for the luminosity of the sun (not a bad assumption)?



There are two neutrinos + 26.7 MeV per Helium nucleus. The power is given by

$$\mathcal{L} = \frac{3.86x10^{26}Js^{-1}}{1.602x10^{-12}JMeV^{-1}} = 2.4x10^{39}MeVs^{-1} \quad (68)$$

The number of Helium nuclei produced is

$$No.(He) = \frac{2.4x10^{39}Hes^{-1}}{26.7MeV} \quad (69)$$

and the number of neutrinos is twice this. Therefore, the number of neutrinos intersecting the earth is

$$\nu/m^2/s = \frac{1.8x10^{38}s^{-1}}{4\pi r^2_{earth-sun}m^2} = 6.37x10^{14}m^{-2}s^{-1} \quad (70)$$

This is a large number! The neutrinos are mostly very low energy but they travel a very long way and actually produce a very good experimental arena to look for neutrino oscillations. There are several experiments which have reported many fewer neutrinos from the sun than expected from the Standard Solar Model. This is thought to be further evidence for neutrino oscillations. There is another explanation. Can you think of one?

Figure 14 shows the relative cross-sections (in arbitrary units) for the d-d and the d-t reactions as a function of energy. There is a resonance of an excited state of ${}^5_2$He at about 10MeV which increases the d-t reaction probability and it is this which experimenters have been trying to take advantage of at JET for example in Culham near Oxford. The disadvantage is that tritium must be manufactured and has no natural abundance because of its beta decay with a half life of 17.7 years. In order to produce the temperatures necessary to reach an energy of 60 keV (where the d-t cross section is a maximum) it is necessary to make a **plasma**. This is a very hot gas which is essentially an ion gas because all the electrons have been boiled off their respective nuclei. These temperatures are very hard to sustain and would melt any container which tried to confine the gas. Therefore a pulsed device is used which heats the plasma for short bursts and at the same time the plasma is confined with a pulsed magnetic field (this, at least, cannot burn!) A second method called inertial confinement has also been attempted. This confines the gas in mm sized balls which are pulsed by laser beams to heat them up under pressure. The engineering problems associated with controlled fusion are immense and a sustained fusion has not yet been achieved.

to achieve temperature T in a d-t plasma, the energy that must be input to the plasma is given by:

$$E_{IN} = 4\rho_d(3k_BT/2)/\text{unitvolume} \quad (71)$$

where $\rho_d$ is the density of deuterium and tritium ions ($\rho_d = \rho_t$). The electron density is $2\rho_d$ and therefore there are $4\rho_d$particles/unit volume. The reaction rate in the plasma is

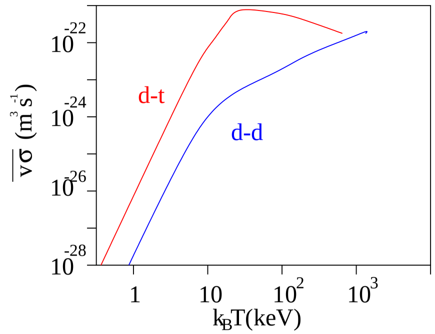



Figure 14: Relative cross-sections for d-d and d-t fusion

$\rho_d^2\bar{\sigma v}$. If the plasma is confined for time $t_c$, then the ratio of (fusion energy out)/(energy out), is given by

$$\frac{\rho_d^2\bar{\sigma v}t_c(17.6MeV)}{6\rho_d k_B T} \quad (72)$$
$$\approx 10^{-19}m^3s^{-1}\rho_d t_c \quad (73)$$
$$(74)$$

and this is clearly an inefficient process. A 'useful' device would need (fusion energy out)/(energy out)>1, and for this to be true,

$$\rho_d t_c > 10^{19}m^{-3}s$$

that is, the number density has to be very high, or the length of time the confinement lasts must be long. This is known as the *Lawson Criterion*.

# 29 LECTURE 29

## 29.1 Nucleosynthesis in Stars

When all the Hydrogen in a star is 'burned', the star will contract. The increased pressure caused by this contraction under gravity will increase the temperature until the Helium is ignited. After the Helium, further stages of nuclear burning set in until Iron and Nickel are formed. At each stage, the higher temperature needed to overcome the higher Coulomb barrier is provided by gravity. Most of the energy, however, is released in the hydrogen burning to Helium, in fact a massive 7.1MeV/nucleon. There is only a further 1.7MeV per nucleon to be released in the complete burning to Iron. Eventually the star becomes a neutron star and a supernova explosion occurs. It is thought that during neutron star formation the heavy elements such as Uranium are formed.



## 29.2 Resonance Enhanced Neutron Capture for Waste Transmutation

We have seen that one of the failings of the nuclear reactor as a power generator is the very long lived radioactive waste which must be stored for many years away from civilization. The idea of Resonance Enhanced Neutron Capture for Waste Transmutation is to bombard radioactive waste with neutrons produced in an accelerator to turn transuranic elements(TUR) and long lived Fission Fragments(FF) into less toxic and more importantly, stable elements. This process makes use of the resonant capture by FF and TUR at certain specific neutron energies.

The radioactive waste is made up of TUR(1%), FF(4%) and Fuel Cladding(95%). TUR is the most dangerous, but this can be induced to fission and used in the reactor itself. The FF can be separated into stable, short and long lived parts (for example $^{90}$Sr and $^{137}$Cs have a half life of about 30 years). The fuel caldding does not present a danger and can be disposed of simply.

In order to create the exactly correct energy for the neutrons for a given resonant capture, the choice of the medium which the FF is prepared in is of paramount importance. The medium must be transparent to neutrons (i.e. cannot absorb neutrons itself) but must be able to slow the neutrons to the correct energy by either:

- undergoing many inelastic scatters such as happens in $D_2O$ or graphite which quickly makes the neutrons thermal (energies of 1eV or less)
- undergoing many *elastic* scatters with the nuclei in the medium of Pb or some other heavy nuclei which slowly reduces the neutron energy by a given amount per collision and thus sweeps out the entire energy spectrum from original energy to thermal

It is indeed fortuitous that adding an extra neutron to a long lived FF can turn it into either a short lived isotope or even a completely stable element.

We can look at one example: that of $^{99}$Tc (Technetium) which is one of the worst offenders of radioactive waste. Technetium is soluble in water with a half life of $10\dot{1}0^5$ and is retained in the stomach,blood, saliva and the thyroid gland. This obviously presents a hazard to the biological life in the vicinity of the stored nuclear waste, especially if it is under water, but $^{99}$Tc has a large resonant cross section for neutron capture leading to the completely stable isotope $^{100}$Ru. The energies for resonant neutron capture are shown in Figure 15.

The number of neutrons needed to convert the huge amounts of radioactive waste is very large and the best sources of neutrons are either those from a spallation source (this is where protons produced in an accelerator knock out neutrons from a particular target) or from reactors themselves. However, because the neutron energy must be carefully matched, the waste products must be prepared in their appropriate medium and cannot be just left in the fuel rods to capture the neutrons. It is estimated that the power requirements to convert the waste would be about 10% of the electricity produced by a light water reactor.

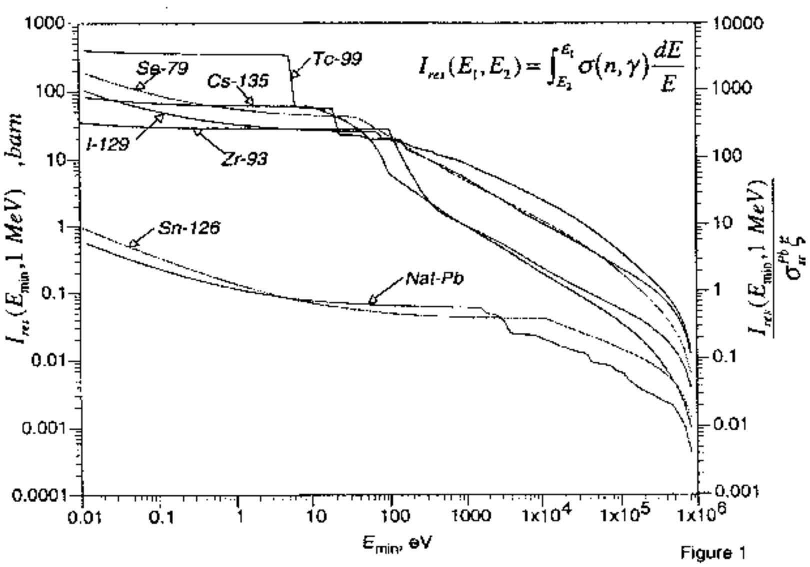



Figure 15: resonance integral for elements produced in reactors. A sudden drop in $I_{res}$ indicates a resonant capture at a particular neutron energy.



This is obviously a great idea, but the equipment is expensive and so would require a certain capital investment before use could be made of a parasitic waste transmutor running off the reactor itself. Furthermore, the engineering is in a very early stage. This must represent a huge growth area in the next decade.

# 30 LECTURE 30

## 30.1 The Nuclear Shell Model

The basic premise of the shell model of nuclei is that a single nucleon moves in a common potential $V(r)$ which is a combination of the effect of all the other nucleons and that the interaction between nucleons can be viewed as a small perturbation. This allows for the possibility that nucleons act as if they are bound in angular momentum shells around a central potential, much like electrons are bound around the central nucleus in an atom. The central potential in the case of nuclei is not an identifiable object but just the effective potential of all the nucleons. The striking evidence which forced this view of the nucleus to the forefront was the existence of **magic numbers**. We have seen that nuclei with even numbers of nucleons are more stable than those with an odd number. It was found that special numbers of protons or neutrons form particularly stable configurations. These are when either Z or N is equal to:

2,8,14,20,28,50,82,126

and these numbers are called the magic numbers. There is a plethora of different evidence which backs up this claim that magic numbers do indeed exist, but it is not really relevant to the discussion. For more information about this see Williams.

The shell models' validity is very questionable and so it is rather surprising (and so far unexplained) that this model can be used successfully to explain the magic number phenomena and also more detailed properties such as the spins, magnetic moments and level spectra of many nuclei.

Start by assuming that the common potential $V(r)$ is an oscillator potential of the form $V=-V_0+ar^2$. The levels of this potential are

(1s),(2p),(3d)=(2s),(4f)=(3p),(5g)=(4d)=(3s)......

and each group of degenerate levels is a shell. Figure 16 shows the effective potential $V_{eff}$:

$$V_{eff}=-V_0+cr^2+\frac{l(l+1)}{2mr^2} \tag{75}$$

and the states below $V_{eff}$=0 are bound states. The protons and neutrons live *independently* from each other and so there are two separate shell structures, one for neutrons and one for protons. There is only this shell structure because protons and neutrons are spin $\frac{1}{2}$ and therefore obey Fermi-Dirac statistics.

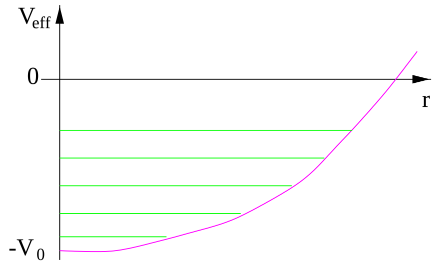



Figure 16: Effective potential for the nuclear shell model

| Oscillator shells | l | $l_j$ | from shell | to shell |
|---|---|---|---|---|
| I | 0 | $s_{\frac{1}{2}}$ | 2 | 2 |
| II | 1 | $p_{\frac{3}{2}}, p_{\frac{1}{2}}$ | 6 | 8 |
| IIa | 2,0 | $d_{\frac{5}{2}}$ | 6 | 14 |
| III | 2,0 | $d_{\frac{3}{2}}, s_{\frac{1}{2}}$ | 6 | 20 |
| IIIa | 3,1 | $f_{\frac{7}{2}}$ | 8 | 28 |
| IV | 3,1 | $f_{\frac{5}{2}}, p_{\frac{3}{2}}, p_{\frac{1}{2}}, g_{\frac{9}{2}}$ | 22 | 50 |
| V | 4,2,0 | $g_{\frac{7}{2}}, d_{\frac{5}{2}}, d_{\frac{3}{2}}, s_{\frac{1}{2}}, h_{\frac{11}{2}}$ | 32 | 82 |
| VI | 5,3,1 | $h_{\frac{9}{2}}, f_{\frac{7}{2}}, f_{\frac{5}{2}, p_{\frac{3}{2}}, p_{\frac{1}{2}}}$ | 44 | 126 |

Table 2: : Oscillator Shells

In order to predict the correct magic numbers, it is necessary to take into account the spin-orbit coupling. It is J which is conserved:

$$J^2 = (\bar{L} + \bar{S})^2 \quad (76)$$

$$= \bar{L}^2 + \bar{L}^2 + 2\bar{L} \cdot \bar{S} \quad (77)$$